MA827

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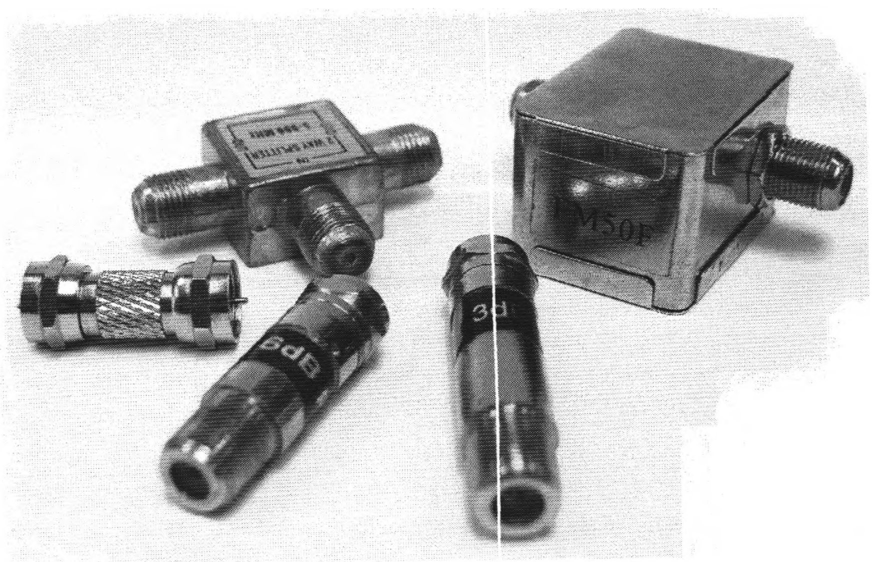
AC Principles and Applications
Part 2



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AC Principles and Applications - Part 2

Version 2.0 - June 2022

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Ultimo NSW 2007

Contents

Introduction

This workbook is the second of two that have been written to help students satisfy the assessment requirements for the following Units:

- UEEEC0074 - Troubleshoot resonance circuits in an electronics apparatus

More information about electrotechnology training packages and the above Units can be found at www.training.gov.au/home/tga. Type the course/unit number in the "Quick search" and press return.

Textbooks

The following textbooks can be used to supplement the notes in this workbook. Most (if not all) of these books are available from the library in Building D.

- Donovan (2002). *Electronics Mathematics* (2nd edition). Prentice Hall.
- Edwards & Myer (1993). *Electronics; a basic course* (2nd edition). McGraw Hill.
- Fiore (2001). *Op amps and linear integrated circuits*. Thompson.
- Floyd (2000). *Principles of Electric Circuits* (6th edition). Prentice Hall.
- Floyd (1999). *Electronic Devices* (5th edition). Prentice Hall.
- Gates (2001). *Introduction to electronics* (4th edition). Thompson Printing.
- Boylestad & Nashelsky (1999). *Electronic Devices and Circuit Theory* (7th edition). Prentice Hall.
- Hazen (1991). *Exploring Electronic Devices*. Saunders Publishing.
- Ludeman (1990) *Electronic Devices and Circuits*. Saunders Publishing.
- Malvino (1998). *Electronic Principles* (6th edition). McGraw Hill.
- Metzger (1998). *Electronics Pocket Handbook* (3rd edition). Prentice Hall.
- Paynter & Boydell (2002). *Electronics Technology Fundamentals*. Prentice Hall.

Lesson organiser

The following is a suggestion only. It may be changed to meet student needs and/or different modes of delivery.

Lesson	Section	Topics and activities
11	Pt 2 - 1	■ **Possible assessment - Refer to the UAG for details** ■ Complex waveforms and an introduction to the spectrum analyser
12	Pt 2 - 2	■ An introduction to filters ■ Practise identifying filter responses using an oscilloscope
13	Pt 2 - 3	■ Filter performance characteristics ■ Practise determining the frequency performance of filters using an oscilloscope
14	Pt 2 - 4	■ Low-pass and high-pass filter circuits ■ Practise measuring the output voltage of, and the phase shift introduced by, LPFs and HPFs using an oscilloscope
15	Pt 2 - 5	■ Resonance ■ Practise measuring the voltage across components in resonant circuits using an oscilloscope
16	Pt 2 - 6	■ Band-pass and band-stop filter circuits ■ Practise measuring the output voltage of, and the phase shift introduced by, BPFs and BSFs using an oscilloscope
17	■ **Possible assessment - Refer to the UAG for details**	
18	■ **Possible assessment - Refer to the UAG for details**	

Student notes

Section 1 Complex waveforms and an introduction to the spectrum analyser

Purpose To develop your understanding of complex waveforms and the spectrum analyser.

Objectives At the end of this section you should be able to:

- Define the terms *complex waveform*, *fundamental frequency*, *harmonics* and *harmonic distortion*
- Recognise a complex waveform
- Describe the difference between repetitive waveforms represented in the time and frequency domains
- List the frequency components present in a squarewave given its frequency
- State the usefulness of the square wave as a test signal for audio systems
- Explain the difference between signals shown in the time and frequency domains
- Name the test equipment that visually represent signals in the time and frequency domains

Introduction

You've probably noticed that, when testing the operation of analog circuits such as amplifiers, sinewaves are often used for the input signal. This is convenient because it allows us to concentrate on the circuit's basic operation without complication. However, signals in real electronic equipment are rarely pure sinewaves. They are much more likely to look like some of the signals shown in Figure 1 below.

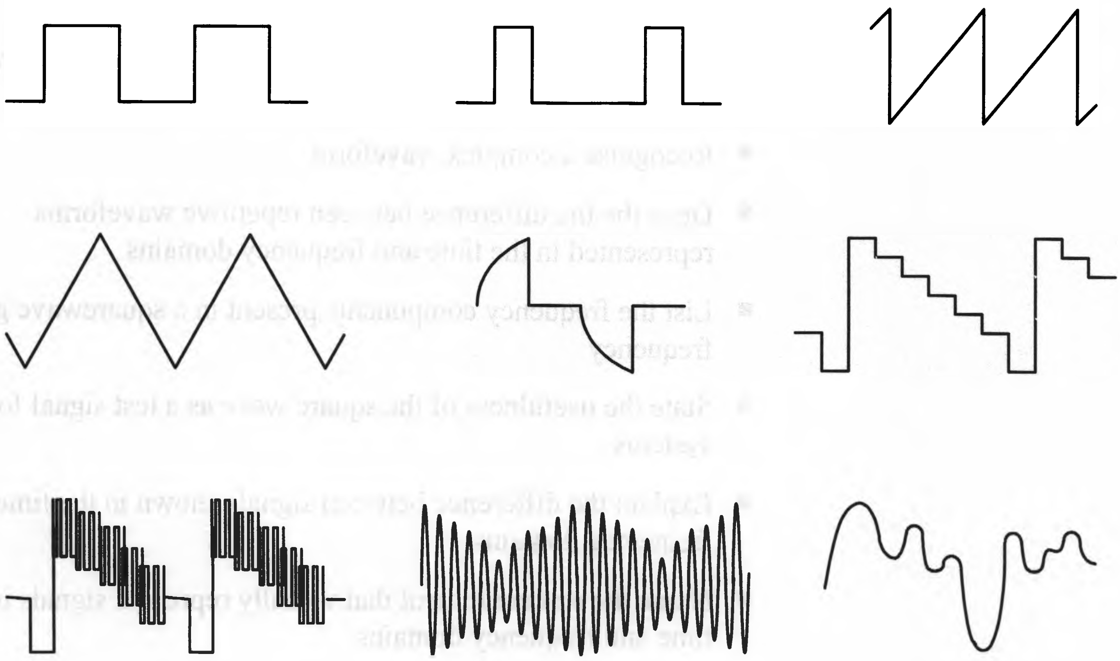

Figure 1 *Signals that are commonly found in electronic equipment*

These signals are all far more complicated than pure sinewaves - both visually and electrically - and for this reason they're grouped into a category of signals called *complex waveforms*.

Complex waveforms

The term *complex waveform* can be defined as "any signal that is not a pure sinewave". This definition means that even a sinewave with a DC component (or *DC offset*) is a complex waveform.

Importantly, every complex waveform (including those shown in Figure 1) can be shown to be made up of a number of pure sinewaves (and often a DC component) added together. This might seem ridiculous when you think of how different the waveshapes are but consider the example in Figure 2 below. Two sinewaves with different amplitudes and frequencies are shown and beneath them is the result of simply adding them together.

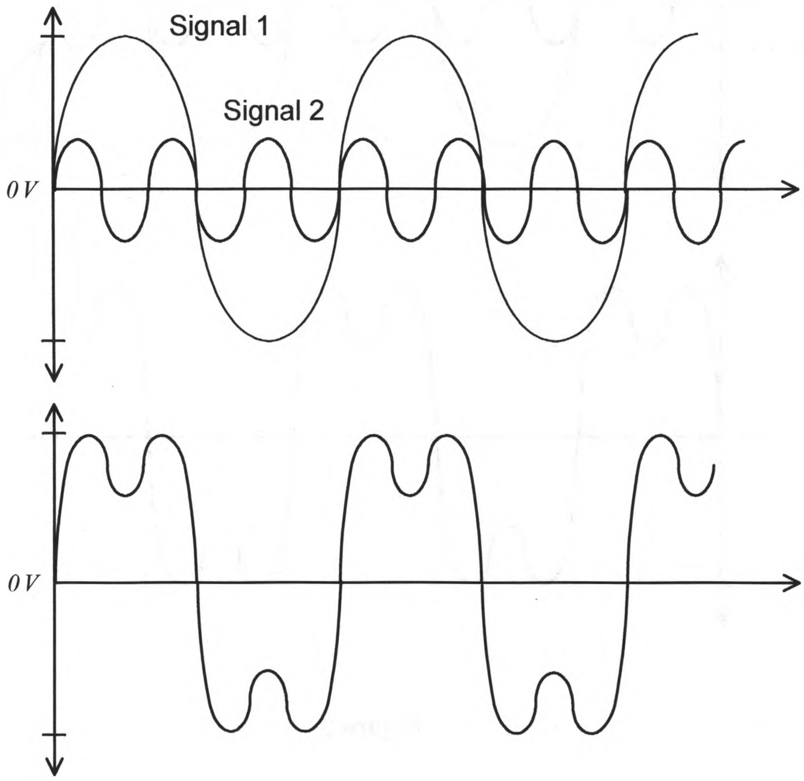

Figure 2 *The result of the addition of two pure sinewaves*

The result is a waveshape that is almost nothing like a sinewave. Some very different waveshapes can be produced by the two sinewaves if either their phase relationship, frequency and/or their peak-to-peak voltages are changed. Adding extra sinewaves can produce even more complex waveshapes.

Regardless of their shape, complex waveform all have a *fundamental frequency* plus *harmonics*. The fundamental frequency is the sinewave that has the same frequency of the complex waveform itself. The harmonics are all the other sinewaves. In the example of the complex waveform on the previous page (redrawn below) it is made up of the fundamental and one harmonic.

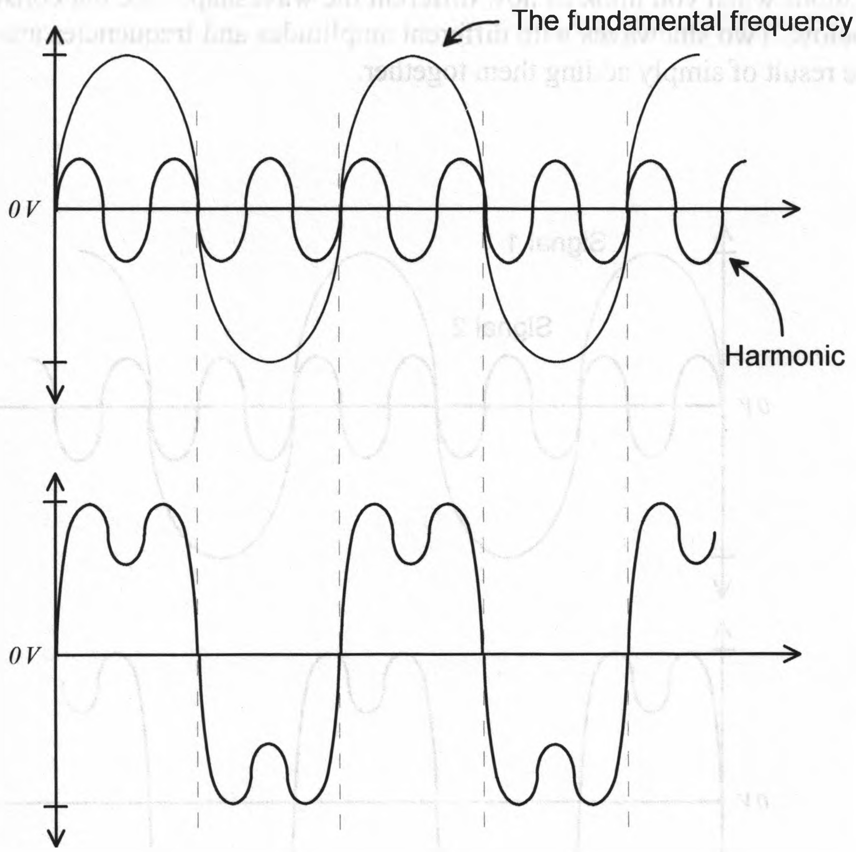

Figure 3

Harmonics with frequencies that are even multiples of the fundamental frequency are called *even harmonics*. For example, if the fundamental frequency of a complex waveform is 1kHz then even harmonics include: 2kHz, 4kHz, 6kHz, 8kHz and so on. Harmonics with frequencies that are odd multiples of the fundamental frequency are called *odd harmonics*. Therefore, odd harmonics of a complex waveform with a 1kHz fundamental include: 3kHz, 5kHz, 7kHz and so on.

The harmonic content of squarewaves

The ideal square wave is made up of the fundamental frequency and an infinite number of odd harmonics that are all in phase with each other. The voltage of each harmonic is proportionally smaller than the fundamental's peak-to-peak voltage by the reciprocal of the harmonic's number.

This is best understood by considering an example. The table below shows the harmonic content of a 1kHz 10Vpp squarewave.

Frequency components	Electrical properties	
Fundamental	1kHz	10Vpp*
Third harmonic	3kHz	3.33Vpp
Fifth harmonic	5kHz	2Vpp
Seventh harmonic	7kHz	1.43Vpp
Ninth harmonic	9kHz	1.11Vpp
Eleventh harmonic	11kHz	0.91Vpp
Thirteenth harmonic	13kHz	0.77Vpp

. . . and so on to infinity.

Importantly, not every odd harmonic is needed to make a signal that approximates an ideal square wave. For example, Figure 4 below shows the waveform produced by adding only the 3rd, 5th, 7th, 9th, 11th and 13th harmonics at the appropriate voltages. Naturally, the more odd harmonics there are in the waveform, the more it will look like a square wave.

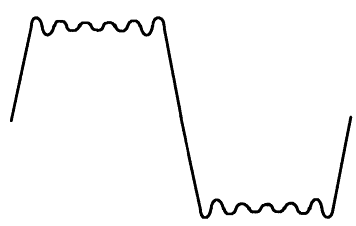

Figure 4 *A complex waveform consisting of the fundamental and odd harmonics to the 13th*

* In actual fact, the fundamental's amplitude is 1.27 times bigger than the squarewave's amplitude with the harmonics being proportions of this value. However, this simplification is sufficient for our purposes here.

The fact that squarewaves are actually complex waveforms consisting entirely of sinewaves (the fundamental and all the odd harmonics) is very useful to know. It means that squarewaves can be used for qualitative testing of the frequency performance of amplifiers, filters and many other electronics systems.

To explain, an ideal squarewave (that is, one consisting of all or most of its harmonics) has very sharp corners. When an ideal squarewave is passed through a system the frequency performance limitations of the system cause some of the harmonics to be lost. This must change the shape of the squarewave at the output of the system. Low to mid-level loss of harmonics (particularly the high frequency harmonics) results in the squarewave having rounded corners as shown in Figure 5 below.

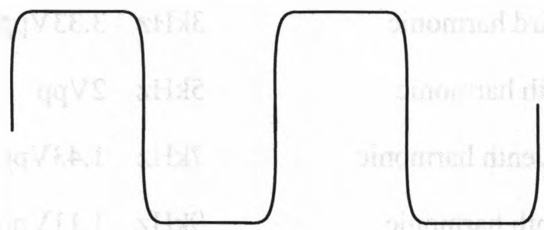

Figure 5 *A squarewave with rounded corners*

Importantly, the greater the rounding of the corners, the greater the loss of harmonics. That being the case, if a squarewave is connected to the input of a system under test and the resulting output signal has more rounding than normal then we know that there is something wrong with the system's frequency response.

The time and frequency domains

All of the AC waveforms you have learnt about in this subject so far have been drawn in what is called the *time domain.* That is, the X-axis of the graph represents time (while the Y-axis represents voltage or current). Figure 6 below is shown to emphasise this point and this is how AC waveforms are displayed by oscilloscopes. Recall that the settings on the oscilloscope's horizontal sweep control (or timebase) are seconds per division (or ms/div and μs/div).

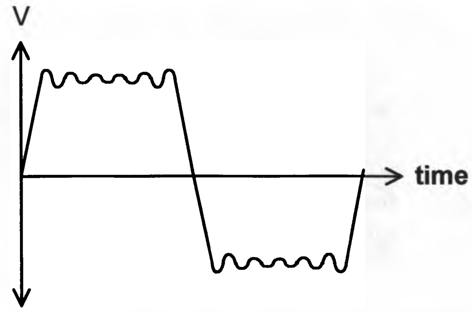

Figure 6

As Figure 6 is the same as Figure 4 on page 1-5, we know that it is made up of the fundamental frequency plus odd harmonics to the thirteenth harmonic. Yet, the individual components that make it up are not visible on a time domain graph or on an oscilloscope. However, they are visible on a frequency domain graph where the horizontal axis represents frequency instead of time. The signal in Figure 6 above is shown in the frequency domain graph in Figure 7 below.

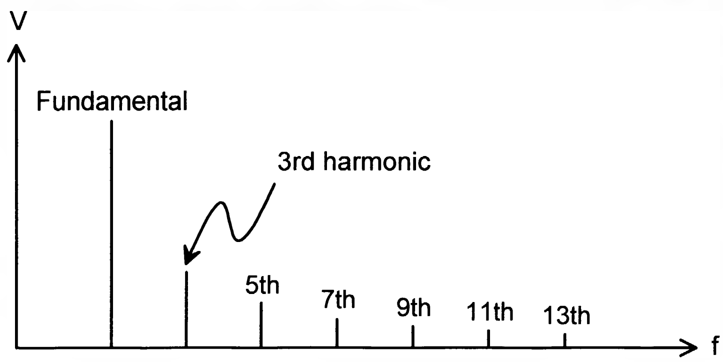

Figure 7 *The frequency domain graph of the waveshape in Figure 6 above*

The spectrum analyser

The *spectrum analyser* is a sophisticated piece of test equipment that, at first glance, looks a little like an oscilloscope. However, they're different to oscilloscopes in that they show signals in the frequency domain instead of the time domain. In other words, when operated, they produce displays like Figure 7 and not Figure 6. Figure 8 below shows an example of a spectrum analyser.

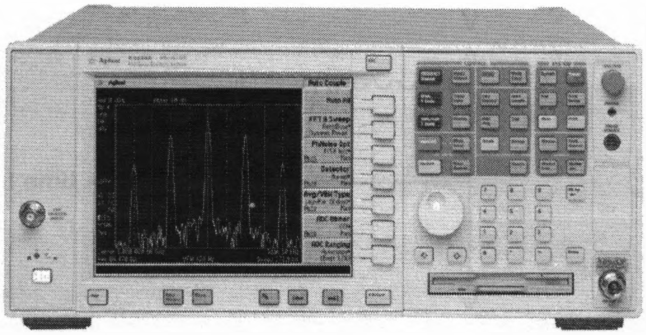

Figure 8 *A spectrum analyser*

Spectrum analysers are far less common than oscilloscopes because they're much more expensive. And, they're not usually needed for day-to-day repairs of domestic electronics equipment so it's a bit hard for smaller workshops to justify purchasing one. However, they are used extensively in analog and digital communications environments for looking at the spectrum of radio signals. If you're working in the communications or telecommunications industry, your workshop may have one.

Part of the reason spectrum analysers are so expensive is due to the difficulty of the job they're required to do. They must analyse the input signal and determine how many sinewaves make it up, what their frequencies are and what their amplitudes are. This can be done using filters (which you'll learn about in the next lesson) or using a mathematical process based on calculus called *Fast Fourier Transform* (or just FFT). Either way, the electronics are sophisticated and costly.

The Electronics Department of Sydney Institute has three spectrum analysers all of which can be difficult to operate even for experienced technicians. As such, your teacher will demonstrate the operation of a spectrum analyser in lieu of a skill practice exercise.

Demonstration

This will involve the teacher demonstrating the operation of one of the department's spectrum analysers with the following test signals.

- A (near) pure sinewave from an Audio Signal Generator
- The sinewave from one of the bench-mounted Function Generators
- A squarewave from one of the bench-mounted Function Generators
- A triangular wave from one of the bench-mounted Function Generators
- An AM signal with a 100kHz carrier modulated by a 2kHz sinewave and generated using the Emona Telecommunications Trainer 101 (or just "Biskit")
- An FM signal with a 100kHz carrier modulated by a 2kHz sinewave and generated using the Biskit

Student notes

Review questions

Answer these questions to check your understanding of what you have learnt for this chapter. Doing this will also help to prepare you for the tests.

1. What piece of test equipment displays electrical signals in the frequency domain?

2. What is the frequency and peak-to-peak voltage of the fundamental frequency of a 12kHz 850mVp-p squarewave?

3. What is the frequency and peak-to-peak voltage of the third harmonic of the same signal?

4. What is the frequency and peak-to-peak voltage of the fifth harmonic of the same signal?

5. Draw the frequency domain graph for a 15kHz 3Vp-p square wave indicating all voltage and frequency information up to and including the 11th harmonic.

Student notes

Section 2 An introduction to filters

Purpose

To develop your understanding of the need for filtering in electronic systems and to develop your ability to recognise the frequency response of filters circuits.

Objectives

At the end of this section you should be able to:

- Explain the purpose of filters in electronic systems
- List practical applications for a low-pass, high-pass, band-pass and band-stop filter
- Draw schematic symbols for a low-pass, high-pass, band-pass and band-stop filter
- Explain the operation of a low-pass, high-pass, band-pass and band-stop filter module with respect to their frequency response
- Draw the ideal and practical frequency response of a low-pass, high-pass, band-pass and band-stop filter
- Use an oscilloscope to perform simple frequency sweep analysis to experimentally determine a filter's type
- Use filtering to experimentally prove the spectral composition of a square wave

Introduction

Section 1 of this workbook introduced you to the fact that all non-sinusoidal waveforms are actually made up of two or more sinewaves added together and probably with a DC component as well.

In electronics systems like communications equipment, the sinewaves that make up complex waveforms often need to be separated. For example, the signal in an audio amplifier may look like Figure 1 which would contain sinewaves ranging from about 20Hz to 20kHz. However, it is common in hi-fi systems to separate the signal so that low frequencies are sent to the *woofer* (a low frequency speaker), high frequencies sent to the *tweeter* (a high frequency speaker) and the frequencies in between are sent to the mid-range speaker.

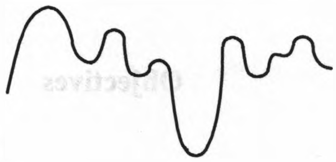

Figure 1

AM radios receive signals like that shown in Figure 2. Such a signal might contain thousands of sinewaves used for the voice and music information "mixed" with a carrier signal so they can be transmitted. At the receiver, the voice and music frequencies must be recovered so they can be listened to.

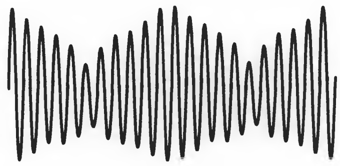

Figure 2

Similarly, televisions receive signals like that shown in Figure 3 which might contain millions of sinewaves used for voice, music and video. For this signal to be turned back into pictures and sound the sinewaves that make it up must be separated and sent to the different sections inside the tv.

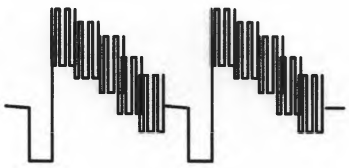

Figure 3

To separate the sinewaves that make up these and other complex waveforms we use circuits called *filters*.

What is a filter?

A filter is a circuit that is designed to allow some sinewaves to pass through from input to output relatively unaffected but *attenuate* (reduce the size of) others. This is analogous to a water filter which lets water pass through but catches or "filters out" impurities such as microbes leaving only clean water (ideally).

The basic filter types

Filters can be categorised by their frequency response. That is, they are grouped by the types of frequencies they pass and stop. There are four basic filter responses:

- Low-pass
- High-pass
- Band-pass
- Band-stop

Let's consider the four types in more detail.

The low-pass filter

Schematic symbols

Figure 4 shows three commonly used schematic symbols for low-pass filters.

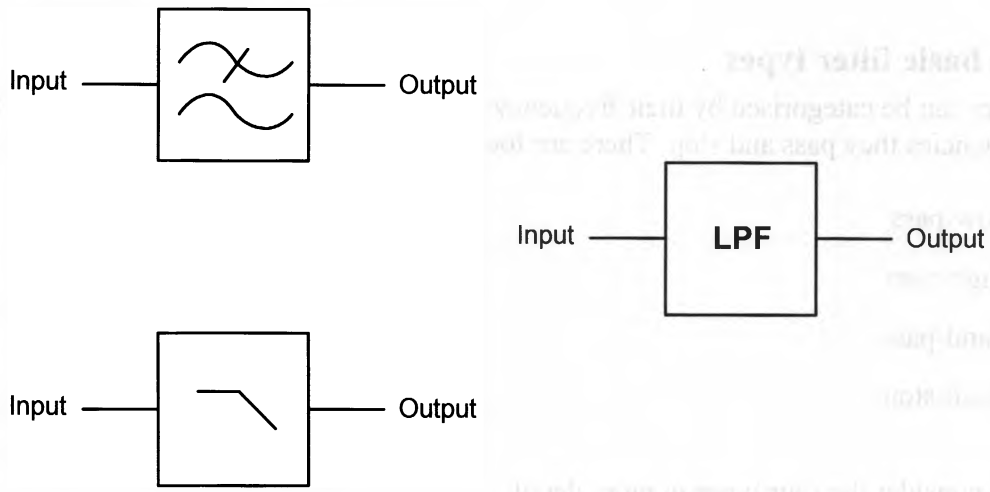

Figure 4 *Commonly used symbols for low-pass filters*

Frequency response

Low-pass filters allow sinewaves below a set frequency to pass from input to output relatively unaffected. Sinewaves at frequencies above the set frequency are attenuated or, ideally, blocked altogether. The frequency response of an ideal low-pass filter is shown in Figure 5 below.

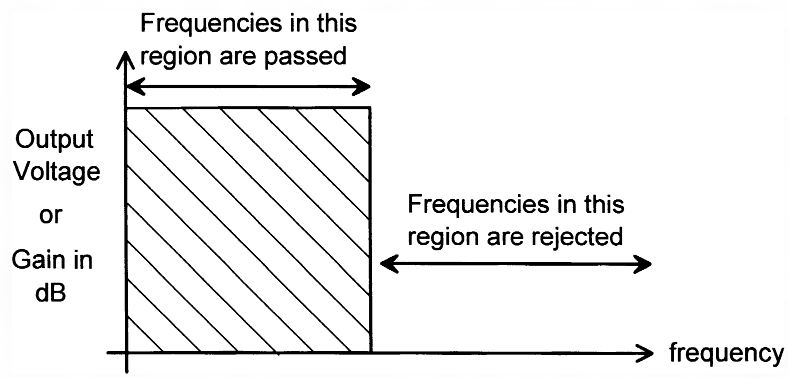

Figure 5 *The frequency response of an ideal low-pass filter*

Frequency response graphs can be difficult to interpret at first so let's look at the operation of a low-pass filter in more detail. Figure 6 shows an ideal low-pass filter that passes all frequencies up to 4kHz. So, if we input a 5Vp-p 1kHz sinewave then we will get a 5Vp-p 1kHz sinewave at the output.

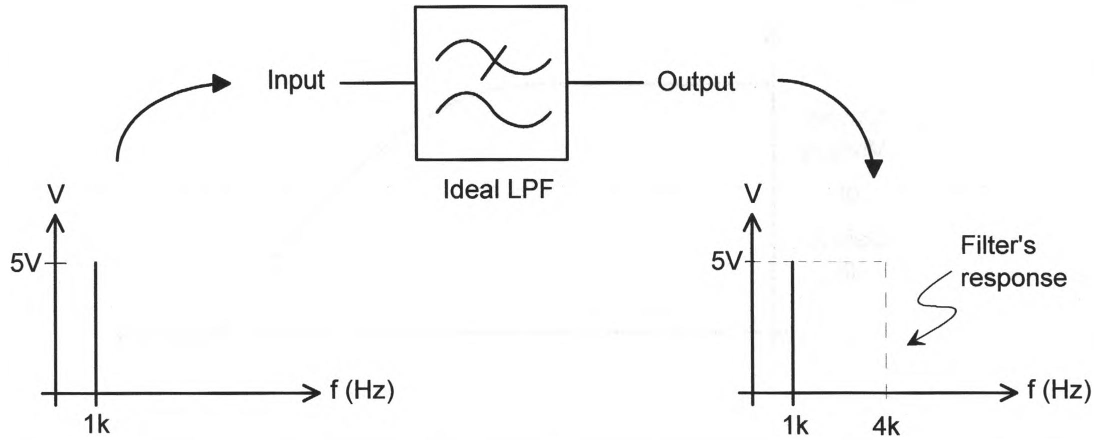

Figure 6 *The frequency domain graphs of the input and output signals in an ideal low-pass filter*

Provided the input signal is less than 4kHz, the output signal will be exactly the same as the input signal. However, if we change the input signal's frequency to anything higher than 4kHz (say 6kHz) then ideally there will not be any voltage at the output of the low-pass filter as shown in see Figure 7 below.

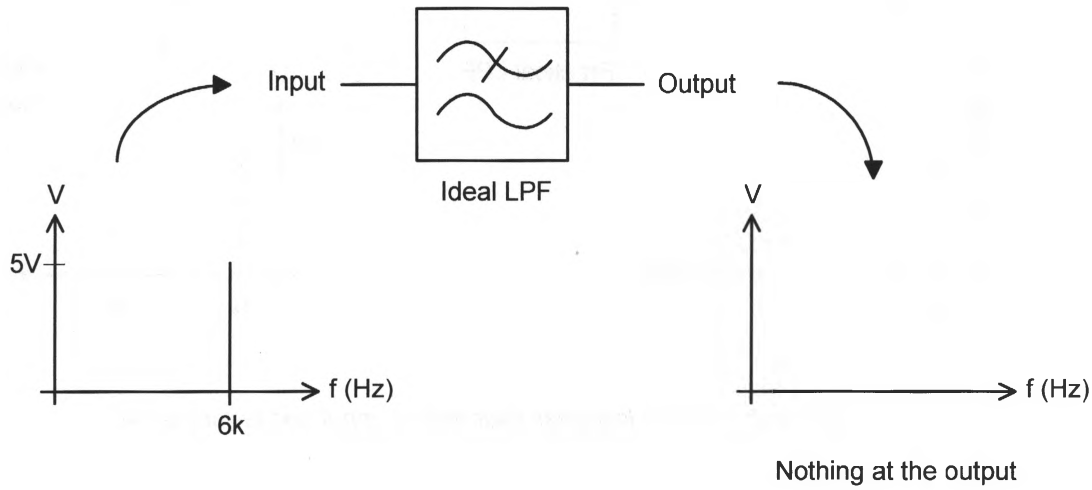

Figure 7 *An ideal low-pass filter with an input signal but no output*

As we know, few things are perfect. Water filters can do a good job of cleaning the water but some particles will always get through. The same is true of electrical filters. They can do a good job of catching the sinewaves at unwanted frequencies but some still get through. The frequency response of a practical low-pass filter is much more likely to look like Figure 8.

Figure 8 *The frequency response of a practical low-pass filter*

Again, to make this a little more meaningful, let's do an example with values. Figure 9 below shows a practical low-pass filter with an upper frequency roll-off point of 4kHz. So, if we input a 5Vp-p 1kHz sinewave then we will get a 5Vp-p 1kHz sinewave at the output.

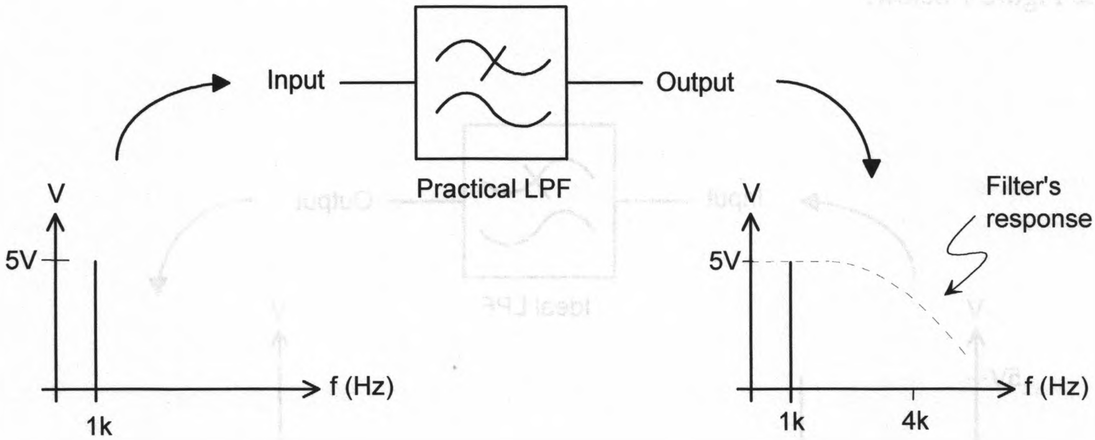

Figure 9 *A practical low-pass filter with an input and output signal*

However, if the input frequency is changed to a 5Vp-p sinewave at 6kHz then, unlike the ideal low-pass filter where there would be no output voltage at all (see Figure 7), there will be a small signal out of the practical filter.

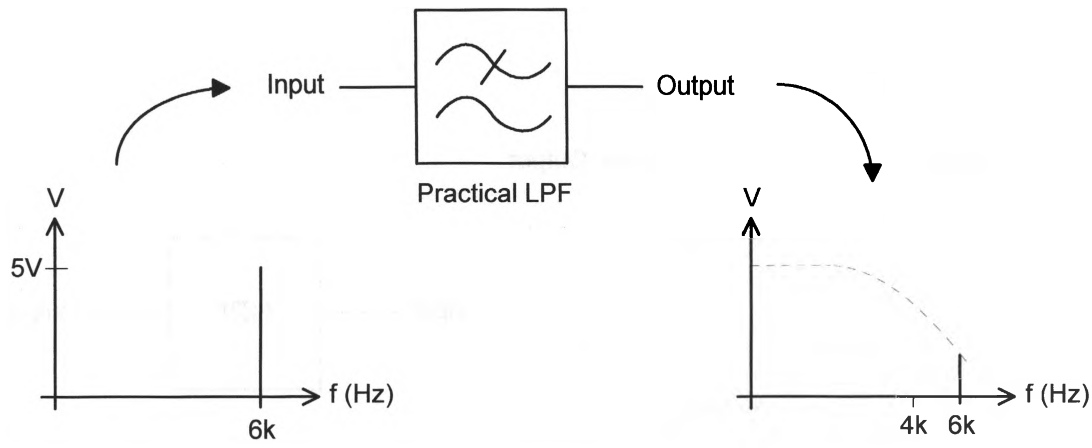

Figure 10 *A practical low-pass filter with an input and output signal*

As the input signal increases in frequency the output voltage will continue to get smaller (attenuate) and can approach zero volts.

Applications
Common applications for low-pass filters include:

- The *crossover* in a speaker box to direct lower frequency signals to the woofer and sub-woofer.
- The bass control circuit in a hi-fi system used to vary the amount of lower frequencies amplified and sent to the speakers.
- In a DC power supply (which you can think of as an AC to DC converter) to smooth the output voltage by filtering out the AC ripple voltage.

The high-pass filter

Schematic symbols

Figure 11 shows three commonly used schematic symbols for high-pass filters.

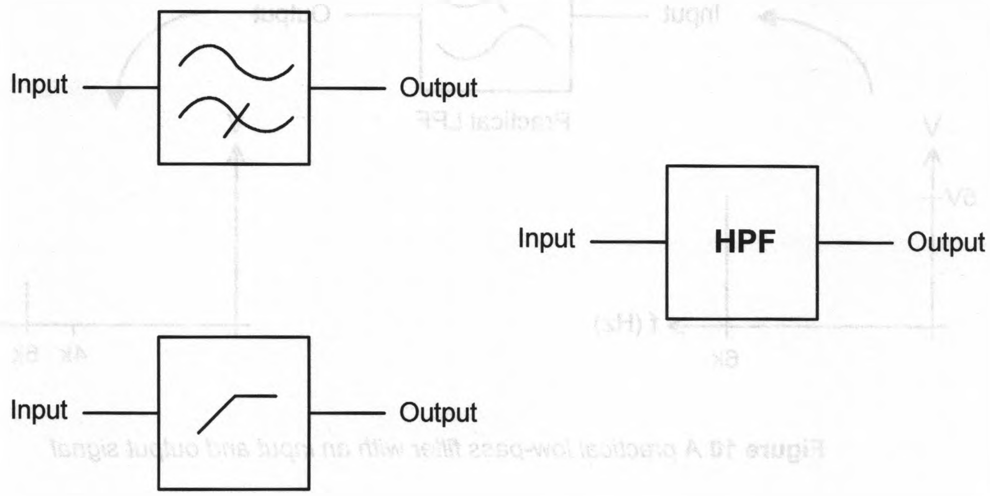

Figure 11 *Commonly used schematic symbols for high-pass filters*

Frequency response

High-pass filters allow sinewaves above a set frequency to pass from input to output relatively unaffected but attenuate sinewaves at frequencies below the set frequency. This is opposite to the operation of low-pass filters. The frequency response of an ideal high-pass filter is shown in Figure 12 below.

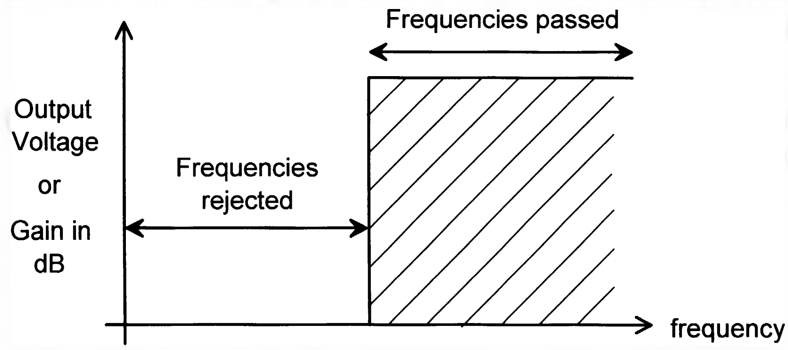

Figure 12 *Frequency response of an ideal high-pass filter*

The frequency response of a practical high pass filter is shown in Figure 13.

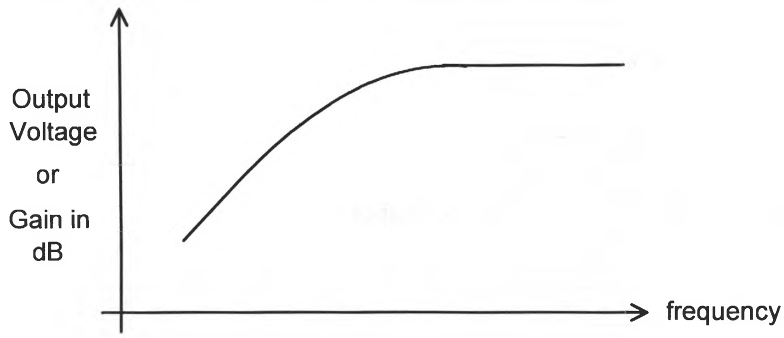

Figure 13 *Frequency response of a practical high-pass filter*

Applications
Common applications for high-pass filters include:

- The *crossover* in a speaker box to direct higher frequency signals to the tweeter.
- The treble control circuit in a hi-fi system used to vary the amount of higher frequencies amplified and sent to the speakers.
- The *rumble filter* (on the mic input of pro-audio gear).

The band-pass filter

Schematic symbols
Figure 14 shows three commonly used schematic symbols for band-pass filters.

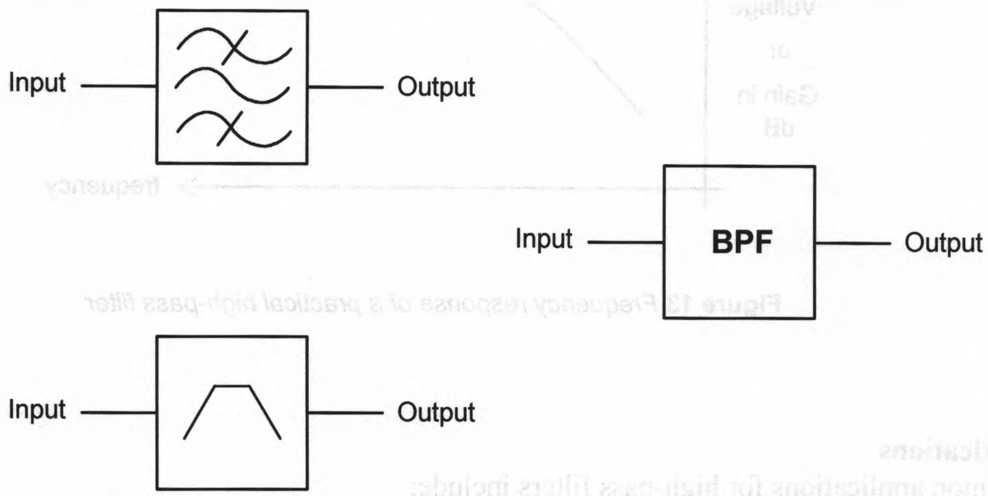

Figure 14 *Commonly used schematic symbols for band-pass filters*

Frequency response
Band-pass filters allow sinewaves between two set frequencies to pass through relatively unaffected but attenuate sinewaves at frequencies above or below this. The frequency response of an ideal band-pass filter is shown in Figure 15 below.

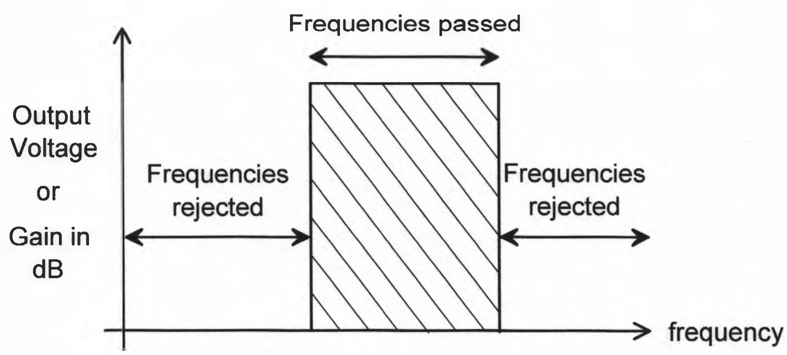

Figure 15 *Frequency response of an ideal band-pass filter*

The frequency response of practical band-pass filters is shown in Figures 16 and 17.

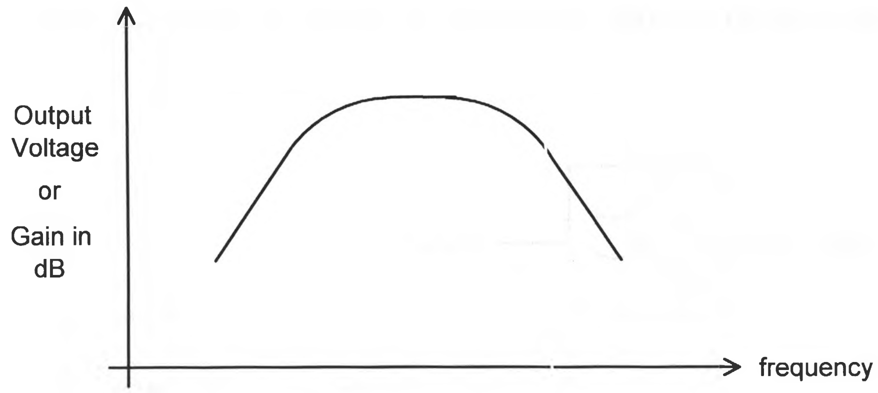

Figure 16 *The frequency response of a practical band-pass filter with a wide bandwidth*

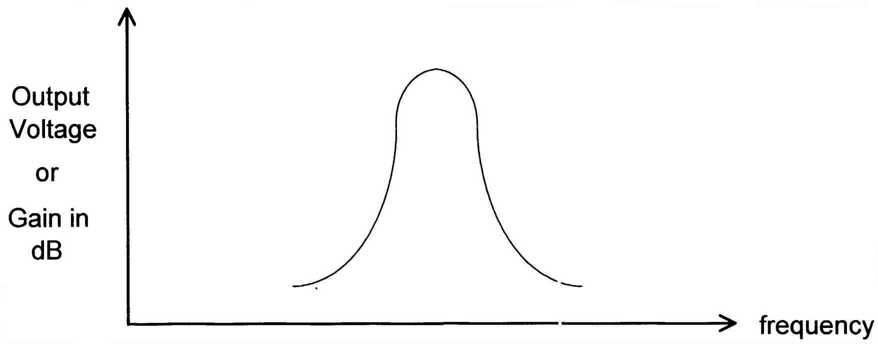

Figure 17 *The frequency response of a practical band-pass filter with a narrow bandwidth*

Both of these graphs show the kind of response that a band-pass filter might have. The differences in bandwidth are due to the filters' design which in turn is determined by its application. Some communication systems require filters with bandwidths as narrow as 17kHz (AM radio) and others with bandwidths as wide as 7MHz (television).

Applications
Common applications for band-pass filters include:

- The *crossover* in a speaker box to direct mid-range frequency signals to the mid-range speaker.

- A *graphic equaliser* which can boost or cut selected bands of frequencies to change the tone.

- Tuning circuits in radio and television to select stations.

The band-stop filter

Schematic symbols

Figure 18 shows three commonly used schematic symbols for band-stop filters.

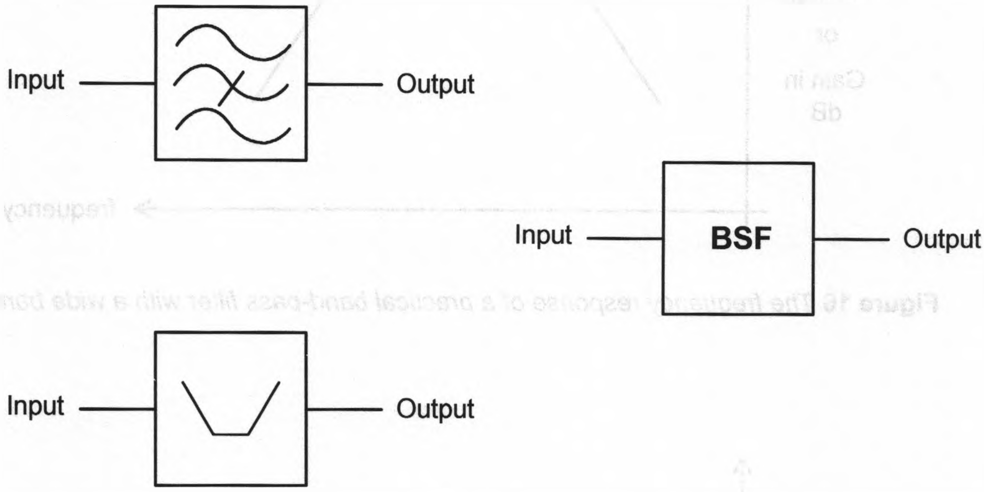

Figure 18 *Commonly used schematic symbols for band-stop filters*

Frequency response

Band-stop filters attenuate sinewaves between two set frequencies but allow sinewaves at frequencies above and below these to pass from input to output relatively unaffected. Band-stop filters are sometimes called *notch* filters because they are said to be cutting out a "notch" in the input's radio frequency spectrum. The frequency response of an ideal band-stop filter is shown in Figure 19 below.

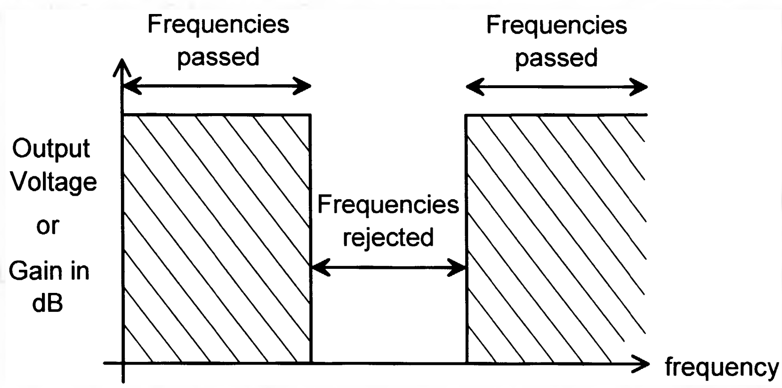

Figure 19 *Frequency response of an ideal band-stop filter*

The frequency response of a practical band-stop filter is shown in Figure 20.

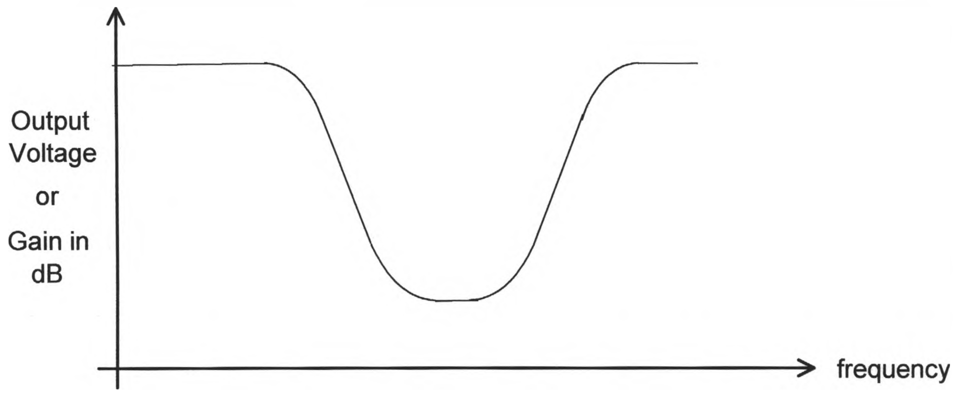

Figure 20 *The frequency response of a practical band-stop filter*

Applications

Common applications for band-stop filters include:

- Scratch filter on a record player.
- More generally, to remove noise at certain frequencies from signals that would interfere with normal operation of communications systems.

Student notes

Skill practice 2

Practise identifying filter responses using the oscilloscope

This exercise is practise for the sorts of skills you may be required to perform in a practical test. Remember, in any practical tests you will be working alone so make sure that you can perform all the steps. It should take you about 1 hour to complete this exercise.

Equipment

- four filter modules (one from each bag)
- three BNC-to-BNC leads

Remember:

Follow TAFE NSW WHS guidance at all times!

Work tasks

1. Read your WHS responsibilities at the top of the form below. Then conduct a WHS risk assessment and record your findings in the space provided.

<table>
<tr><td colspan="2">

Responsibilities of students under the Model WHS Act: s28

- Take reasonable care for your own health and safety by working safely at all times
- Take reasonable care to ensure that your acts or omissions don't put the health and safety of others at risk
- Follow all TAFE NSW WHS guidance and comply with all reasonable instructions from TAFE NSW staff to assist them in complying with the TAFE NSW WHS requirements
- In addition to the above, you must:
  - use and maintain machinery, tools and all other equipment properly and safely
  - ensure that your work area is free of hazards
  - notify a TAFE NSW staff member of actual or potential hazards
  - wear/use prescribed safety equipment
  - take notice of any safety signs and adhere to their instructions

</td></tr>
<tr><td>

Risks involved in this activity include:

Trip hazards (eg students bags)
Objects dropped on feet (while equipment is taken to and from workbenches).

Others: ______________________

____

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Control measures:

Move bags and other objects from walkways
Plan lifting of equipment

Other: ______________________

____

____

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<tr><td colspan="2">

My signature here indicates that I have read and understand my responsibilities under the Model WHS Act s28 (detailed above). I have also conducted a risk assessment before undertaking this activity and have identified measures to control these risks and have implemented them.

Signature: ______________________ Date: ______________________

</td></tr>
</table>

2. Gather the equipment needed for this exercise.

3. Adjust the function generator's output for a 100Hz 5Vpp sinewave.

4. Connect both the function generator's output and the CRO's channel 1 input to the filter module's inputs as shown in Figure 1 below.

5. Connect the CRO's channel 2 input to the filter module's output as shown in Figure 1 below:

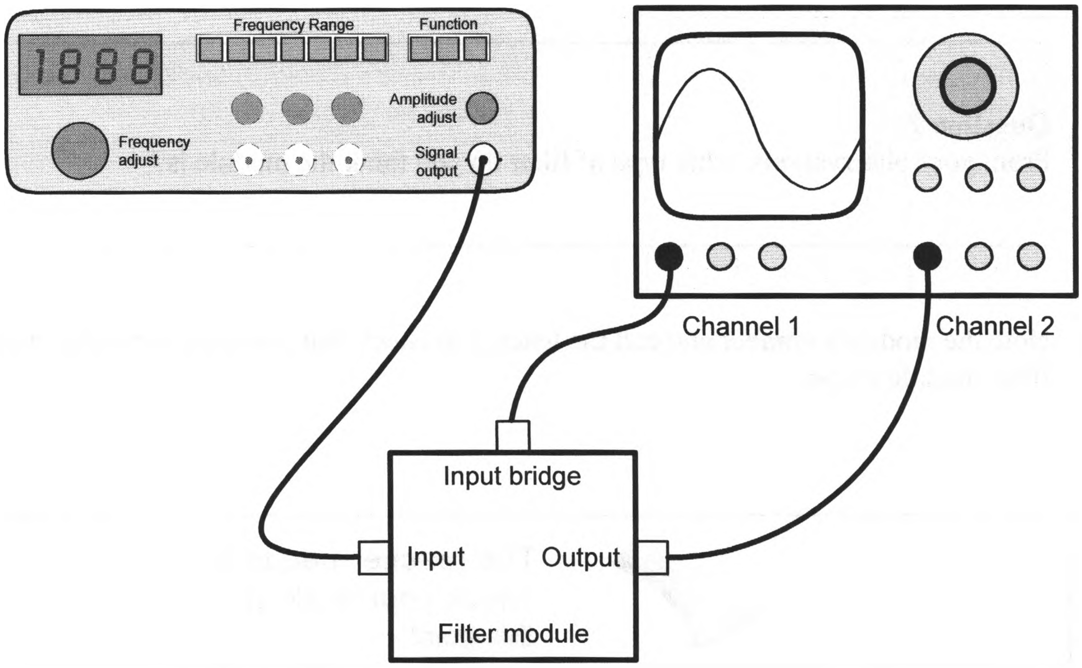


Figure 1

6. Adjust the CRO to view both channels and make a note of the filter's peak-to-peak output voltage. **Note:** Don't be concerned if there isn't an output signal.

7. Slowly increase the frequency of the input signal and observe what happens to the amplitude of the output signal. **Note:** Again, don't be concerned if it doesn't change.

8. Turn the frequency adjust control almost fully anti-clockwise and select the ×1000 frequency range.

9. Repeat Step 6.

10. Turn the frequency adjust control almost fully anti-clockwise again and select the next frequency range.

11. Repeat this test for the remaining ranges on the function generator paying careful attention to what happens to the amplitude of the output voltage as you do.

Question 1
What happened to the amplitude of the output voltage as you swept the input signal from low frequencies to high frequencies?

Question 2
From your observations, what type of filter do you think the module is?

12. Note the module's number and call the teacher to check that you have correctly identified the filter module's type.

The teacher needs to check your work at this point...

13. Repeat steps 3 to 11 for a different filter module.

Question 3
What happened to the amplitude of the output voltage as you swept the input signal from low frequencies to high frequencies?

Question 4
From your observations, what type of filter do you think the module is?

The teacher needs to check your work at this point...

14. Repeat steps 3 to 11 for a different filter module.

Question 5
What happened to the amplitude of the output voltage as you swept the input signal from low frequencies to high frequencies?

Question 6
From your observations, what type of filter do you think the module is?

The teacher needs to check your work at this point...

15. Repeat steps 3 to 11 for the last filter module.

Question 7
What happened to the amplitude of the output voltage as you swept the input signal from low frequencies to high frequencies?

Question 8
From your observations, what type of filter do you think the module is?

____

The teacher needs to check your work at this point...

16. Connect the output of the function generator to the input of the high-pass filter module as well as the CRO's channel 1 input.

17. Connect the CRO's channel 2 input to the output of the high-pass filter module.

18. Adjust the function generator's output for a **100kHz 5Vpp square wave** and draw the shape of the filter's output signal in the space below.

19. Change the frequency of the input signal to 1kHz and draw the output signal's shape.

20. Change the frequency of the input signal to 100Hz and draw the output signal's shape.

Output at 100kHz

Output at 1kHz

Output at 100Hz

Question 9
Why is the output signal's shape different to the input signal?

____

____

____

The teacher needs to check your work at this point...

21. Connect the function generator's output to the input of the low-pass filter module as well as the CRO's channel 1 input.
22. Connect the CRO's channel 2 input to the output of the low-pass filter module.
23. Adjust the function generator's output for a **200Hz 5Vpp square wave** and draw the shape of the filter's output signal in the space below.
24. Change the frequency of the input signal to 2kHz and draw the output signal's shape.
25. Change the frequency of the input signal to 10kHz and draw the output signal's shape.

Output at 200Hz | Output at 2kHz | Output at 10kHz

Question 10
Why is the output signal's shape different to the input signal?

____

____

____

The teacher needs to check your work at this point...

Review questions

Answer these questions to check your understanding of what you have learnt for this chapter. Doing this will also help to prepare you for the tests.

1. What type of filter has the frequency response shown in Figure 1 below?

____

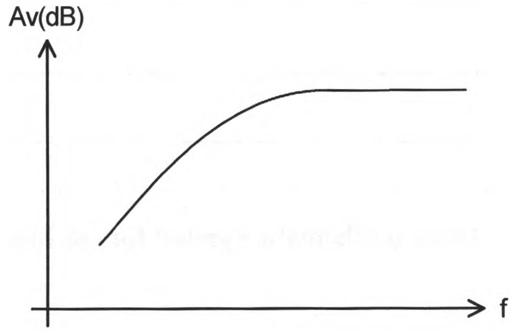


Figure 1

2. What type of filter has the frequency response shown in Figure 2 below?

____

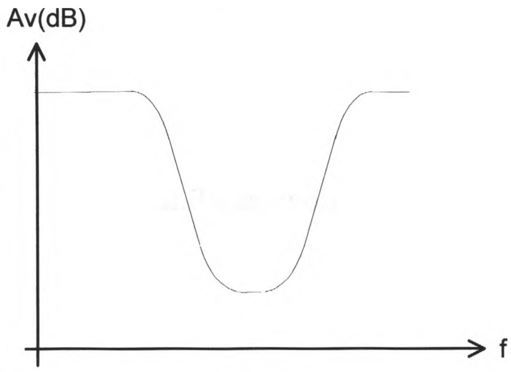


Figure 2

3. What type of filter has the frequency response shown in Figure 3 below?

____

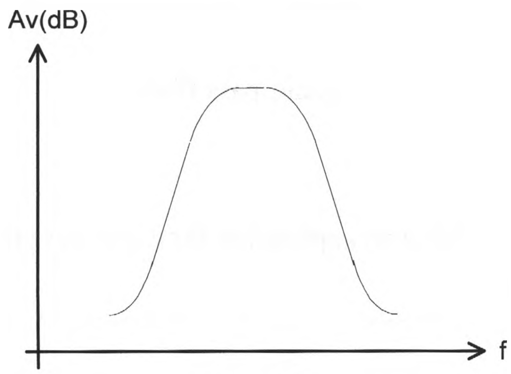


Figure 3

4. What is another name for a band-stop filter?

____

5. Describe the difference between the frequency response of an ideal filter and a practical filter.

____

____

6. Draw a schematic symbol for the four basic types of filter.

Low-pass filter | High-pass filter

Band-pass filter | Band-stop filter

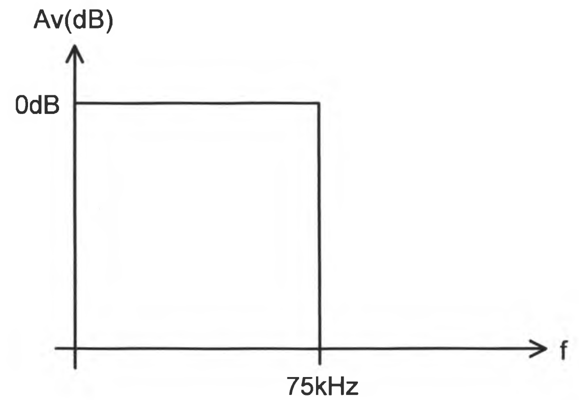
7. Give an application for a low-pass filter.

____

8. What type of filter(s) would a graphic equaliser use?

____

9. What would you expect to see at the output of an ideal low pass filter with the frequency response shown in Figure 4 if a 50kHz 5Vp-p square wave was connected to the input?



____

Figure 4

10. What would you expect at the filter's output if a 100kHz 5Vp-p square wave was connected to the input?

____

Student notes

Section 3 Filter performance characteristics

Purpose

To develop your skills so that you can properly test filter circuits in electronic systems.

Objectives

At the end of this section you should be able to:

- Define terms commonly used to describe the attributes and performance characteristics of filters
- Calculate the output voltage of a filter given the amplitude and frequency of the input voltage, the type of filter, the filter's slope and its insertion loss or amplification
- Use an oscilloscope to measure the gain of filters at a variety of frequencies to determine performance characteristics including the type of filter, cut-off frequency, filter order and pass-band gain

Introduction

Section 1 introduced you to complex waveforms and where they're used. Section 2 introduced you the need for filter circuits and explained the four basic filter-types including the ideal and practical frequency response for each. This section introduces you to some common filter terminology and deepens your understanding of filter performance characteristics so that you're able to predict the approximate output voltage of a filter given specific information about the input signal and the filter's performance.

Filter terminology

There are many features of the frequency response of filters that are of interest to technicians and engineers because they often differ from one filter to another. Of particular interest are: the filter's gain; the roll-off and skirt; the cut-off frequency; the pass-band; the stop-band; the bandwidth; and the centre frequency. You've probably heard of many of the these terms before as they are features of the frequency performance of other electronics equipment such as amplifiers. However, the following notes discuss them all in some detail to make sure you're clear about what they are.

Gain

As you know from your studies of amplifiers, the term *voltage gain* is a measure of the relationship between a circuit's input and output (specifically, $Av = \frac{Vout}{Vin}$). For example, a circuit with a 1V input and a 10V output has a voltage gain of 10. You'll also recall that voltage gain can also be expressed in decibels. This is calculated using the equation $Av_{(dB)} = 20Log\frac{Vout}{Vin}$. A circuit with a gain of 10 also has a gain of 20dB.

Circuits with an output that is the same as the input have a gain of 1 or, when converted to decibels, 0dB.

Not surprisingly, many circuits have an output voltage that is smaller than the input voltage. In other words, they have a loss. However, the comparison of the circuit's input and output voltages can still be called "voltage gain" because it's calculated in the same way ($\frac{Vout}{Vin}$).

The gain of a circuit that has a loss is always a number less than 1. For example, a circuit with a 10V input and a 1V output has a voltage gain of 0.1. Importantly, when a gain of less than one is converted to decibels it gives a negative number. For example, a gain of 0.1 is -20dB.

This is relevant here because, as you'll learn, the relationship between a filter's output compared to its input is calculated in the same way ($\frac{Vout}{Vin}$ and $20Log\frac{Vout}{Vin}$). In other words, we calculate the "voltage gain" of filters even though many filters actually have a loss. On this basis, this workbook standardises on the term "gain" to describe a filter's input-output voltage relationship as it's mathematically correct to do so.

Roll-off and skirt

The *roll-off* is the region of a filter's frequency response where its gain (and output voltage) drops as the frequency changes. The roll-off of low-pass and high-pass filters are shown in Figures 1a and 1b below.

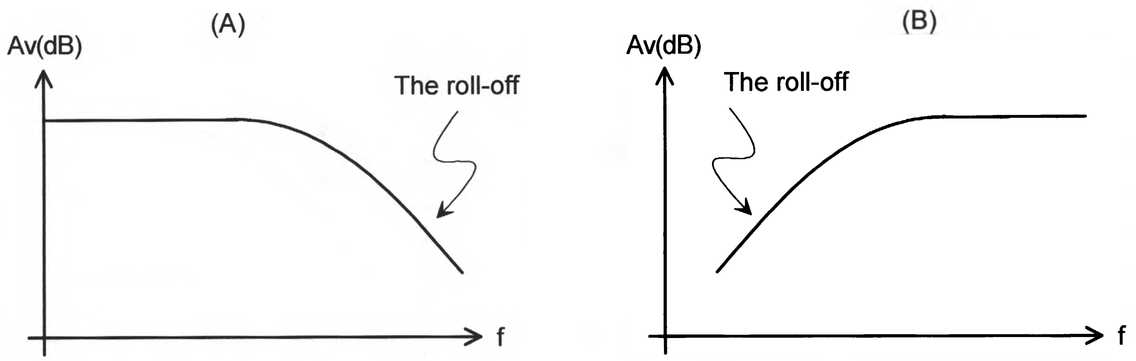


Figure 1 *The roll-off in the frequency response of low-pass and high pass filters*

It is important to remember here that the output voltage is lower in the roll-off region because the filter is filtering out the signals at these frequencies and not because the input signal has got smaller at these frequencies.

Band-pass and band-stop filters exhibit this roll-off effect also. However, the roll-off for these filters is more often referred to as the *skirts* of the response (see Figure 2).

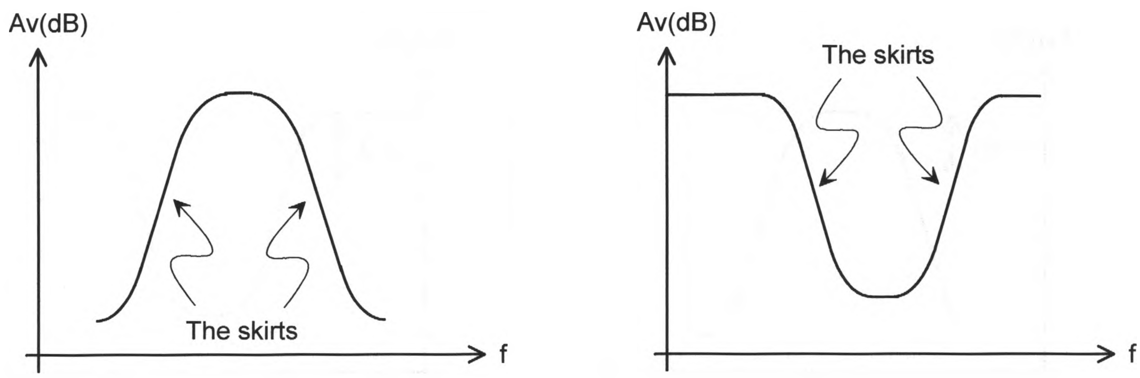


Figure 2 *The skirts on the frequency response of band-pass and band-stop filters*

Cut-off frequency

The *cut-off frequency* (*fc*) is the point in a filter's roll-off where its gain is 70.7% of its maximum (or -3dB lower). Figure 3 shows the cut-off frequency in the response of low-pass and high-pass filters.

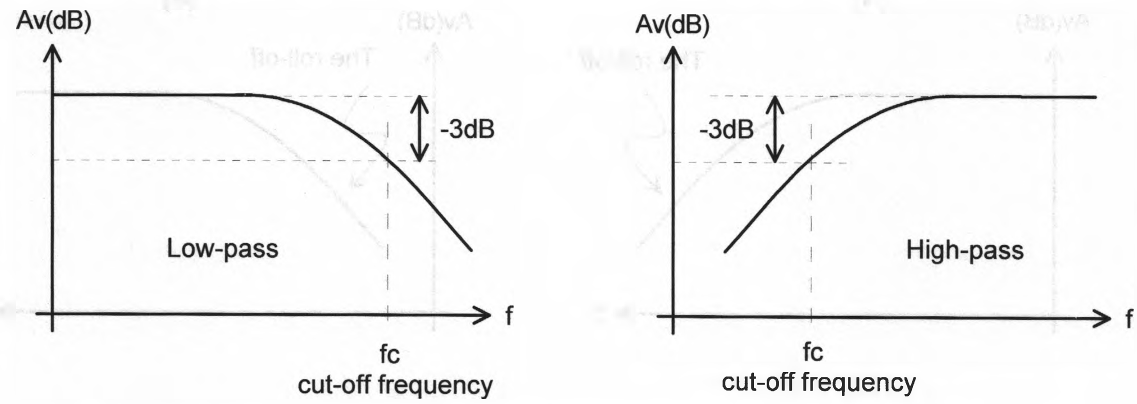


Figure 3 *The cut-off frequency points on low-pass and high-pass filters*

Recall that -3dB is a key figure in terms of power because changes by this amount are noticeable. If the change is less than 3dB we can't hear the difference.

Band-pass and band-stop filters have two cut-off frequencies and as you already know they're called *f1* and *f2* among other things (see Figure 4).

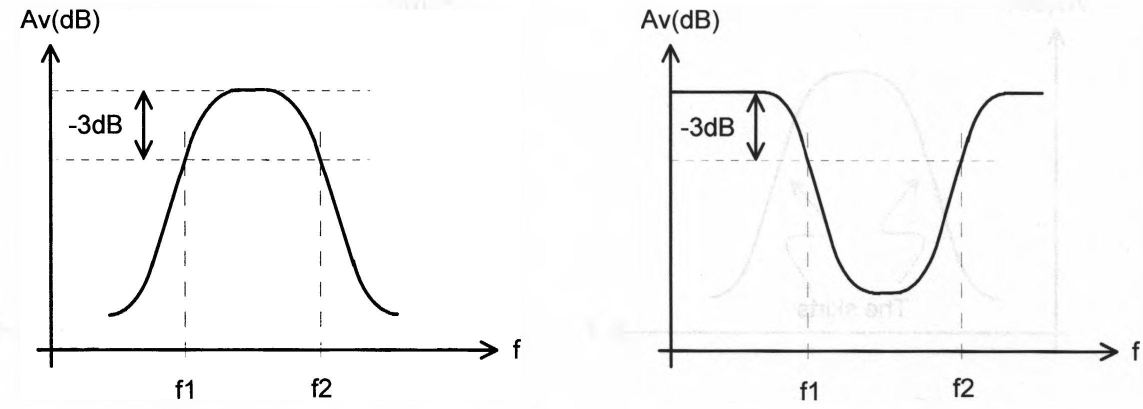


Figure 4 *The f1 and f2 points on band-pass and band-stop filters*

Pass-band

The *pass-band* is the region on a filter's frequency response where its gain (and therefore the output voltage) is maximum and is relatively flat. The pass-bands of low-pass, high-pass and band-pass filters are shown in Figure 5.

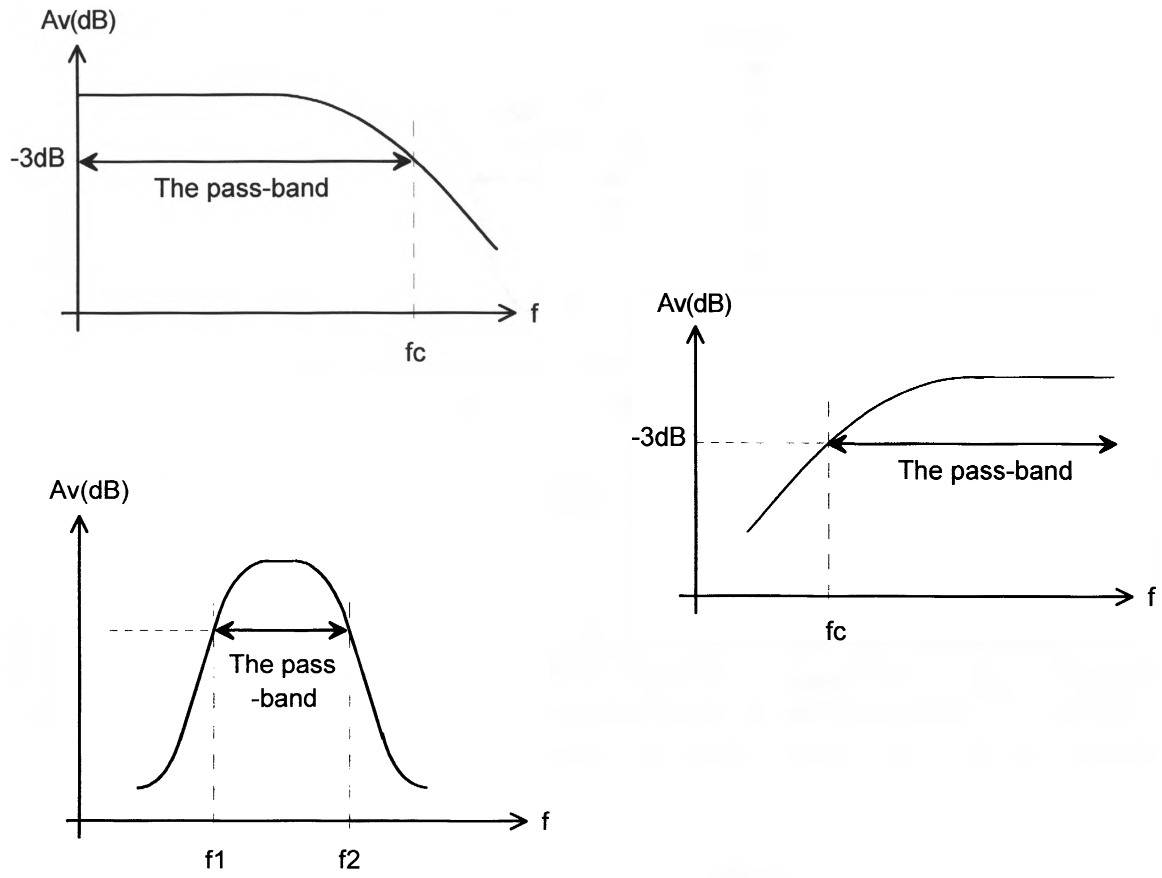


Figure 5 *The pass-band region of low-pass, high-pass and band-pass filters*

Stop-band

The *stop-band* is the region on a band-stop filter's frequency response where its gain (and therefore the output voltage) is attenuated by 3dB or more (relative to the flat part of the response). The stop-band occurs between the *f1* and *f2* points as shown in Figure 6 below.

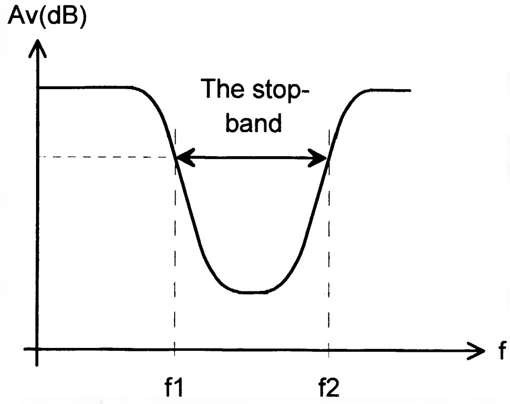


Figure 6 *The stop-band region of band-stop filters*

Bandwidth

The *bandwidth* of a filter describes the frequency width of its passband. However, sometimes the frequency *range* is mistakenly used when referring to the bandwidth. To clarify the difference between range and bandwidth, consider the frequency response of a band-pass filter shown in Figure 7. Its frequency range is 15kHz to 25kHz but its bandwidth is 10kHz.

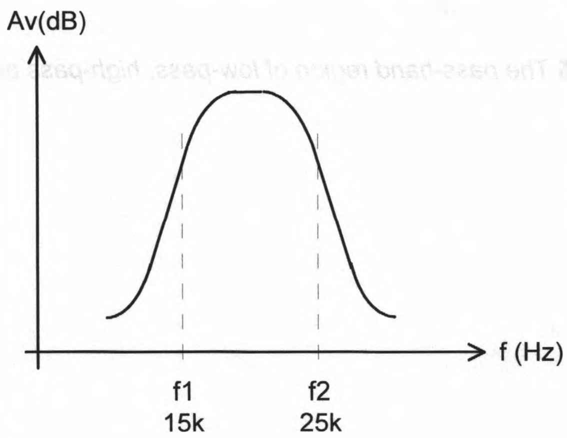


Figure 7

The bandwidth of a band-pass filter is a number that is the difference between the upper and lower roll-off frequencies. This is expressed mathematically as:

$$\boxed{BW = f2 - f1}$$

Centre frequency
The *centre frequency* (*fc*) is the frequency of band-pass filters with the highest gain. It is also the frequency of band-stop filters with the lowest gain. This is shown in Figure 8 below.

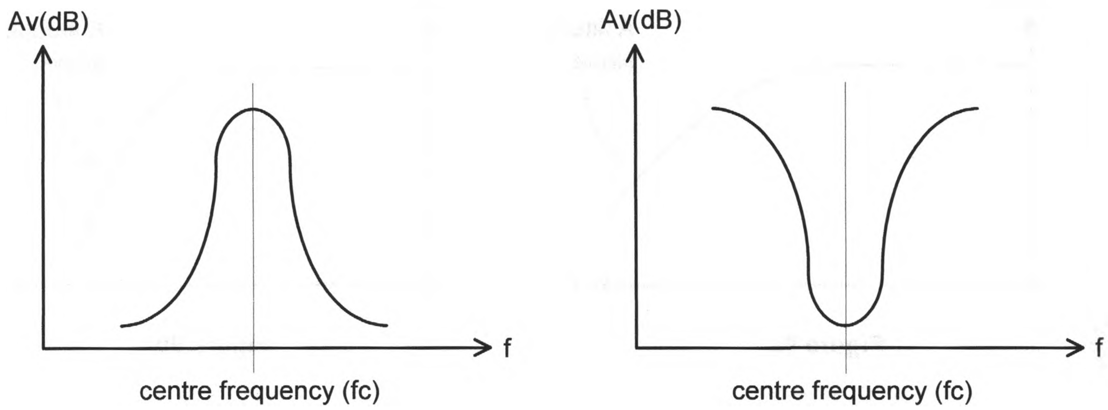


Figure 8 *The centre frequency of band-pass and band-stop filters*

The circuit's centre frequency can be determined from the *f1* and *f2* using: $f_C = \sqrt{f1 \times f2}$ (a calculation known in math as the "geometric mean").

However, if the Q-factor of the band-pass filter is 10 or more then the normal mean (average or what's known in math as the "arithmetic mean") can be used instead for convenience: $f_C = \frac{f1+f2}{2}$. The arithmetic mean is actually an approximation but a good one for filters with a sufficiently high Q-factor. **Note:** Q-factor is explained in Section 5.

Filter performance

Now that some of the terms used to describe the features of filter frequency response have been clarified, we can start to consider some of the important filter characteristics. In particular, technicians should know about: the slope of the roll-off; how to calculate the voltage at the output of a filter at frequencies in the roll-off region; the relationship between filter order and the slope; insertion loss; and pass-band amplification.

Slope

The *slope* of a filter is a measure of the rate at which the filter's gain (and output voltage) drops in the roll-off region. Consider the two low-pass filter responses shown below for example. The slope of the response in Figure 9b is steeper than Figure 9a because the gain drops off at a faster rate.

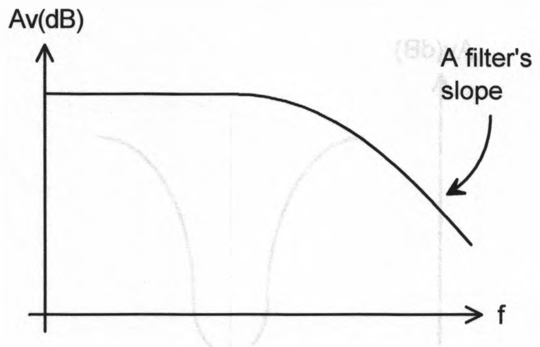


Figure 9a

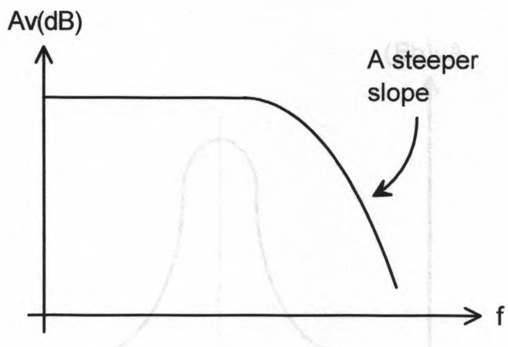


Figure 9b

A filter's slope is actually its gain's "rate of change" with changes in frequency and is commonly expressed in *decibels per octave* (dB/octave). For example, the slope of the response in Figure 9a might be 6dB/octave. As the filter with the response in Figure 9b has a steeper roll-off, it might be something like 12dB/octave.

To be able to interpret these figures, an explanation of the term *octave* is necessary. As you may already know, there are eight notes in the musical scale (in case you haven't seen *The Sound of Music* yet, they're: *doh*, *ray*, *me*, *far*, *so*, *la*, *tee*, *doh*). Each note has a higher frequency than the note before it. Importantly, the difference in frequency between the first and last *doh* is double. For example, if the first *doh* is 1kHz, *ray* will have a higher frequency, *me* will have an even higher frequency and so on with the second *doh* having the highest frequency at 2kHz. In musical terms, the two *dohs* are said to be separated by one octave. That being the case, a change of one octave is equal to a change in frequency of double or half.

That being the case, a filter with a 6dB/octave roll-off has a 6dB change in gain in this region for every doubling (or halving) of the frequency.

This might be better understood with an example. The low-pass filter response in Figure 10 has a slope of 6dB/octave. If the filter's gain at 5kHz is -13dB then its gain at 10kHz (which double the frequency or one octave) is -19dB (6dB lower than -13dB). At 20kHz, the filter's gain is -25dB. At 40kHz the filter's gain is -31dB and so on.

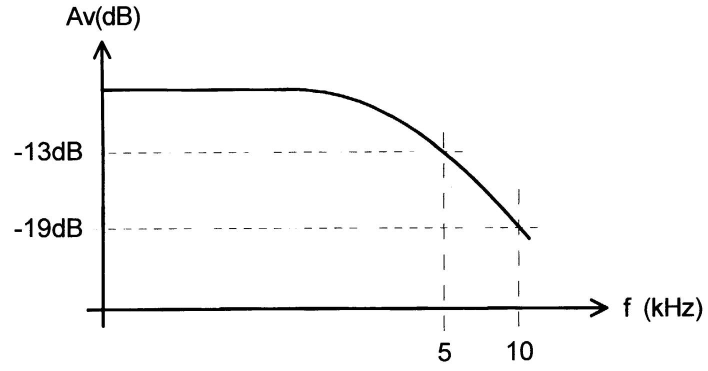


Figure 10

This issue applies equally to other filters. Consider the high-pass filter in Figure 11 below. It has a slope of 6dB/octave and its gain at 22kHz is -9dB. So, at 11kHz (which is half the frequency) the filter's gain is -15dB. At 5.5kHz, the filter's gain is -21dB. At 2.75kHz the filter's gain is -27dB and so on.

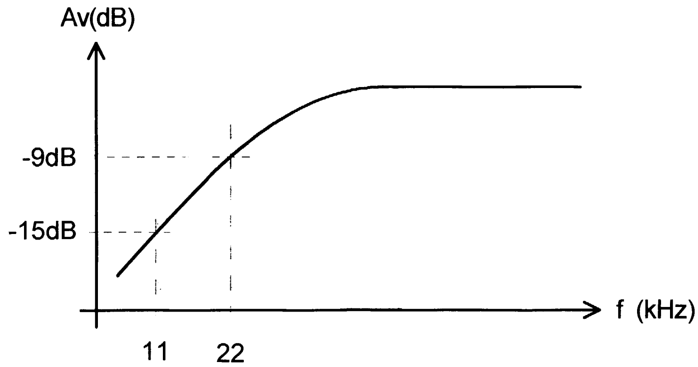


Figure 11

Important note: This discussion is true as long as the reference frequency (that is, the frequency that you start at), has a gain that is equal to or greater than -6dB relative to the filter's pass-band gain. It's only at this point below the filter's maximum output that the line becomes straight again (instead of curvy) indicating a constant rate of change implied by this discussion instead of a changing rate of change. Put another way, if you use the cut-off frequency as the reference frequency, you'll introduce an error of -3dB. That is, the gain at the second frequency will be -3dB smaller than it actually is.

Another common method of specifying the slope is *decibels per decade* (dB/decade). Importantly, a slope of 6dB/octave is the same as 20dB/decade.

Where the octave refers to half or double the frequency, the *decade* refers to one tenth or ten times the frequency. For example, 150kHz is one decade above 15kHz and 1.5MHz is one decade above 150kHz (and two decades above 15kHz). Similarly, 100Hz is one decade below 1kHz and 10Hz is one decade below 100Hz (and two decades lower than 1kHz).

So, if a filter has a slope of 20dB/decade, every time the frequency of an input signal in the roll-off region is increased by ten, the output voltage is 20dB smaller. Again let's demonstrate this using an example. Figure 12 shows a low-pass filter that has a slope of 20dB/decade. At 5kHz the signal gain is -13dB. So at 50kHz (one decade higher) the gain is -33dB, at 500kHz the gain is -53dB and so on.

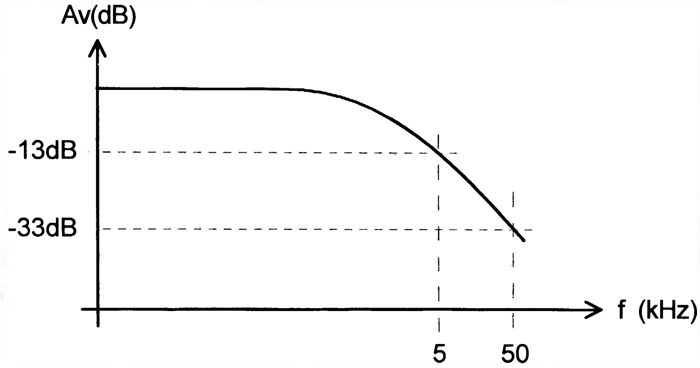


Figure 12

It's possible to convert the slope of the roll-off of a filter from dB/octave to dB/decade and vice versa. You're not expected to be able to do that at trade level. However, you are expected to know that a slope of 6dB/octave is the same as 20dB/decade.

Important note: The note at the bottom of page 3-9 applies equally here.

Calculating the output voltage at frequencies in the roll-off region

Knowing about the slope of the roll-off for a particular filter is actually very handy. It allows you to calculate the output voltage of a filter at various frequencies without having to refer to a graph and without having to use the circuit analysis that you were taught earlier in this subject. You'll find this is especially useful if you're working in the television, communications, telecommunications or audio fields.

To demonstrate this point, consider the filter shown in Figure 13 below. It is a high-pass filter with a slope of 6dB/octave. It has a 5Vp-p sinewave on the input and at 50kHz the gain is -6dB. What would be the output voltage if the input signal's frequency is 25kHz?

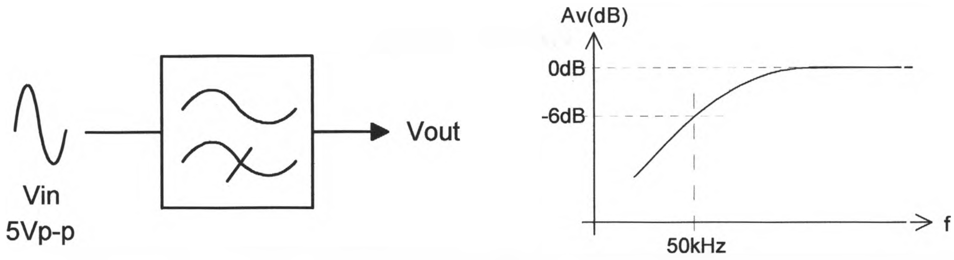


Figure 13

To answer this question, we must first find the filter's gain at 25kHz:

- According to the graph, the filter's gain in the pass-band is 0dB (neither loss nor gain) so this means that at relatively high frequencies the output voltage is the same as the input voltage.
- The graph of the filter's frequency response tells us that at 50kHz the filter's gain is -6dB.
- As the filter's slope is 6dB/octave, its gain at 25kHz (one octave down from *fc*) is -12dB (another 6dB lower than the gain at *fc*).

Once the gain in decibels is known, we can find *Vout* using the equation $Av_{(dB)} = 20Log\frac{Vout}{Vin}$ transposed to make *Vout* the subject:

$$Vout = Vin \times Log^{-1}\left(\frac{Av(dB)}{20}\right)$$

$$Vout = 5Vp\text{-}p \times Log^{-1}\left(\frac{-12dB}{20}\right)$$

$$Vout = 5Vp\text{-}p \times Log^{-1}(-0.6)$$

$$Vout = 5Vp\text{-}p \times 0.2512$$

$$Vout = 1.256Vp\text{-}p$$

Let's do another example. What is the output voltage if the input signal's frequency is 5kHz? First, find the filter's gain at 5kHz:

- We know that the filter's gain at 50kHz is -6dB.
- We want to find the gain at 5kHz which is one decade down from 50kHz.
- The filter's slope of 6dB/octave is also 20dB/decade. So, its gain at 5kHz is 20dB lower than at 50kHz. This means that the filter's gain at 5kHz is -26dB.

So, the output voltage is:

$$Vout = Vin \times Log^{-1}\left(\frac{Av(dB)}{20}\right)$$

$$Vout = 5Vp\text{-}p \times Log^{-1}\left(\frac{-26dB}{20}\right)$$

$$Vout = 5Vp\text{-}p \times Log^{-1}(-1.3)$$

$$Vout = 5Vp\text{-}p \times 0.0501$$

$$Vout = 0.2506Vp\text{-}p$$

Let's do a third example. What is the output voltage if the input signal's frequency is 2.5kHz? Again, the first step is to find the filter's gain at 2.5kHz:

- We know that the filter's gain at 5kHz is -26dB.

- We want to find the gain at 2.5kHz which is one octave down from 5kHz.

- So, the filter's gain at 2.5kHz is 6dB lower than at 5kHz. This means that the filter's gain at 2.5kHz is -32dB.

So, the output voltage is:

$$Vout = Vin \times Log^{-1}\left(\frac{Av(dB)}{20}\right)$$

$$Vout = 5Vp\text{-}p \times Log^{-1}\left(\frac{-32dB}{20}\right)$$

$$Vout = 125.6mVp\text{-}p$$

Practise using this method of finding the output voltage at frequencies in the roll-off region by trying some of the following questions.

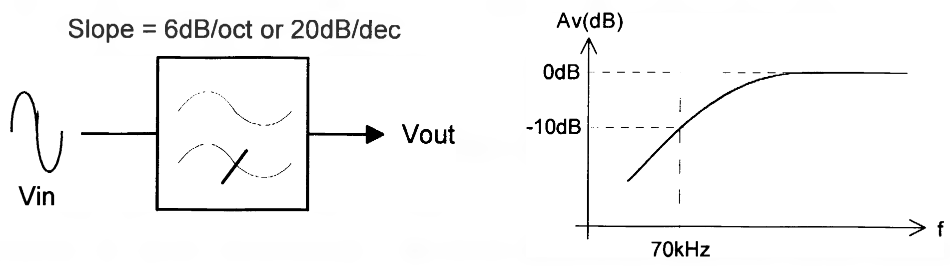


Figure 14

1. What is the gain (in dB) of the filter in Figure 14 at the following frequencies:

(a) 700Hz ______

(b) 17.5kHz ______

(c) 875Hz ______

(d) 350Hz ______

(e) 44Hz ______

2. Calculate the output voltage at 875Hz if the input voltage is 7Vp-p.

Filter order

Although we haven't looked at any filter circuits yet, the simplest designs (which we'll look at in Sections 4 and 6) are made from capacitors and/or inductors. A filter's *order* refers to the number of capacitors and inductors (known in filter terminology as *reactive elements*) that the filter is made from. A first order filter has one reactive element so is made from just one capacitor or one inductor. Examples of first order filters are shown in Figure 15 below (it doesn't matter what type of filters they are at the moment).

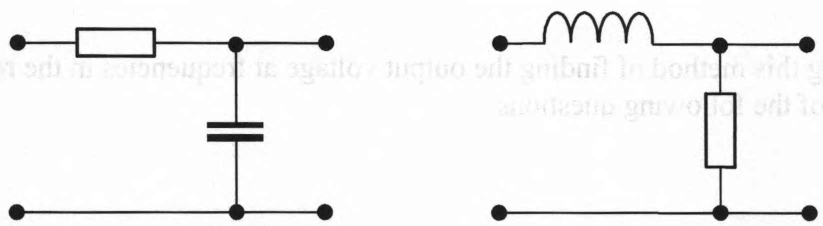


Figure 15 *Examples of first order filters*

Second order filters have two reactive elements so are made from either two capacitors, two inductors or a capacitor and an inductor (see Figure 16). Third order filters have three reactive components and so on.

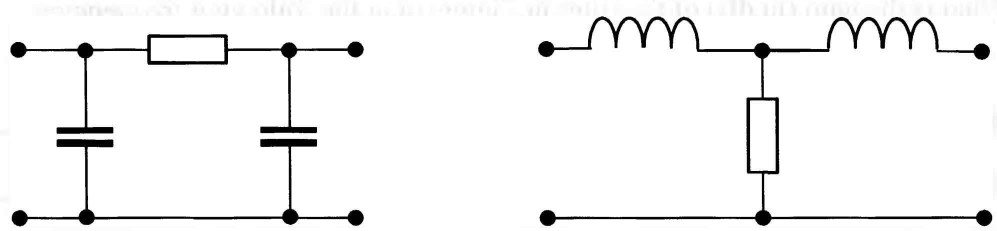


Figure 16 *Examples of second order filters*

A filter's order is important to know because it sets the slope of the roll-off of low-pass and high-pass filters. Table 1 below shows the relationship between filter order and slope. Notice that each additional element in the filter design adds 6dB/octave (or 20dB/decade) to the slope.

Table 1

Filter order	Number of elements	Slope of roll-off	
		dB/octave	dB/decade
1	1	-6	-20
2	2	-12	-40
3	3	-18	-60
4	4	-24	-80

In many applications, the higher the order of a filter the better because the slope is steeper which means that the filter rejects more signals outside the pass-band. However, as usual in electronics there is a trade-off. The greater the (passive) filter's order the greater its insertion loss. [Note: You're only expected to know the roll-off figures of first order filters for the exams.]

Insertion loss

All of the filter frequency response graphs considered so far have shown filters with a pass-band gain of 0dB (neither attenuation nor amplification). This means that, for a 5Vp-p input signal at a frequency in the pass-band, a 5Vp-p signal will appear at the output. However, in practice, many filter designs attenuate signals inside the pass-band. This effect is known as *insertion loss* and is analogous to a water filter that absorbs water as well filtering it so not as much water comes out as goes in.

Figure 17 shows a practical first order low-pass filter with a cut-off frequency of 4kHz. The filter has an insertion loss so, if we input a 5Vp-p sinewave at 1kHz, we don't get 5Vp-p at the output even though 1kHz is in the filter's pass-band.

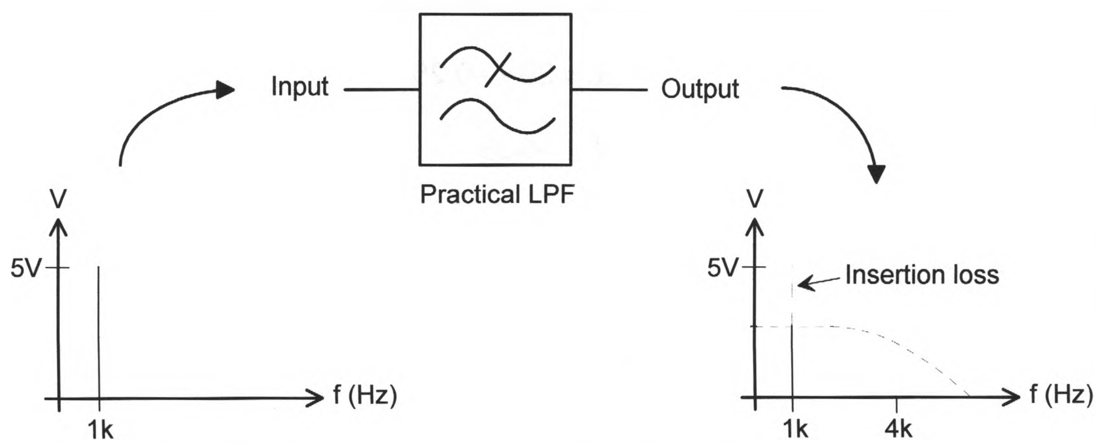


Figure 17 *A practical low-pass filter with an insertion loss*

A filter's insertion loss is usually expressed in decibels and can be found by using the equation:

$$\boxed{\textit{Insertion Loss} = 20\,Log\left(\frac{Vout}{Vin}\right)}$$

Where Vout and Vin are measured at a frequency in the filter's pass-band

Let's do an example with values. What is the insertion loss of the filter in Figure 18?

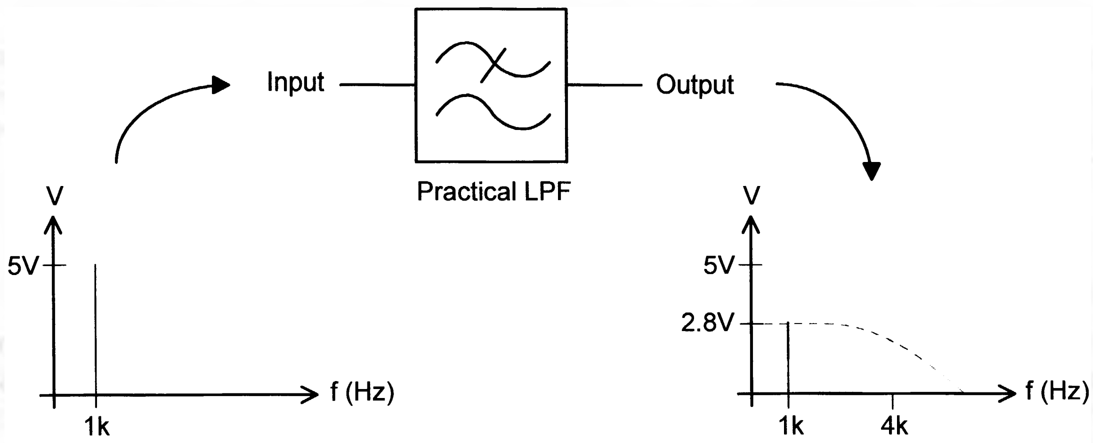


Figure 18

$$IL = 20Log\left(\frac{2.8V}{5V}\right)$$

$$IL = 20Log(0.56)$$

$$IL = 20 \times -0.25$$

$$IL = -5dB$$

Plotted on log/linear graph paper, this filter's response would look like that shown in Figure 19 below.

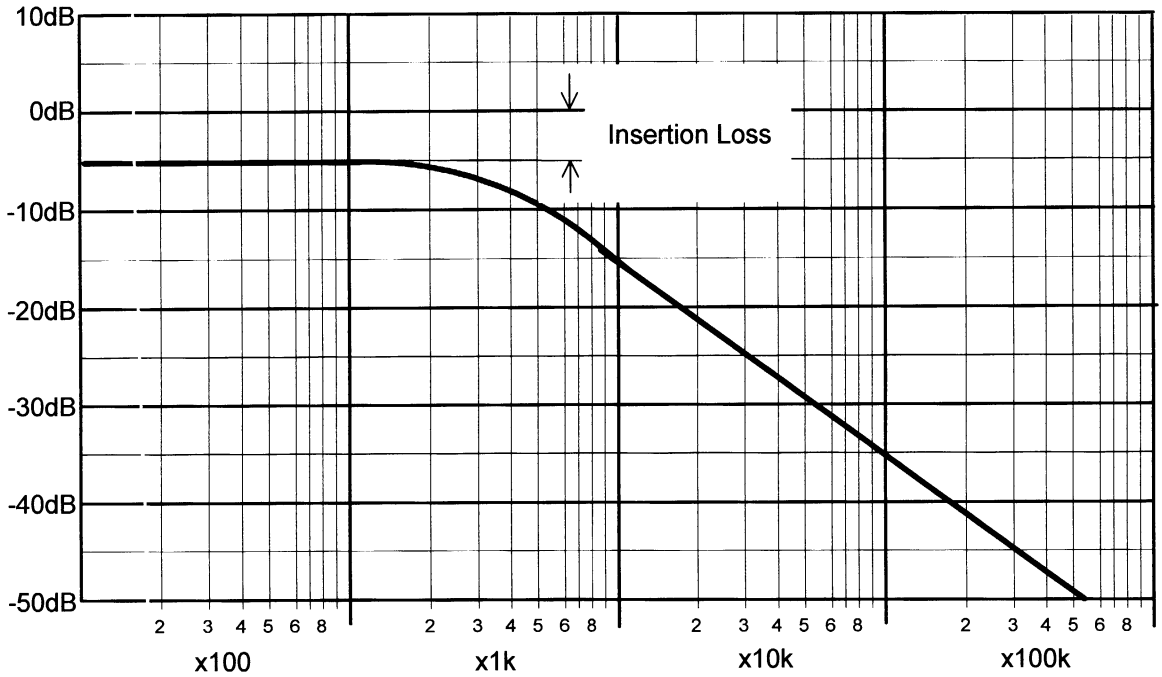


Figure 19 *The frequency response of a first order low-pass filter with a cut-off frequency of 4kHz and a -5dB insertion loss*

A filter's insertion loss doesn't only affect the gain of the filter in its pass-band: It has an effect across the filter's entire frequency response. This means that while the filter's gain at the cut-off frequency (or *f1* & *f2*) is -3dB lower than the pass-band gain, it is -3dB plus the insertion loss lower relative to the input. For example, if a filter has an insertion loss of -5dB its gain at the cut-off frequency is -8dB.

Pass-band amplification

A distinction that is often made between filters is whether they are passive or active. Passive filters are made from only passive components such as resistors, capacitors and inductors (we have already seen examples of these in Figures 15 and 16). Active filters are made using active components such as BJTs, FETs or op amps **as well as** resistors, capacitors and inductors.

The difference between passive and active filters is important. Passive filters tend to have a gain in the pass band of 0dB or less. In other words, the output voltage at frequencies in the pass band is the same size as the input signal or smaller. However, as active filters include components with gain, they may have output voltages greater than the input voltage (even at frequencies in the roll-off).

The frequency response of a first order active filter with a pass-band gain of 15dB looks like the response shown in Figure 21 below.

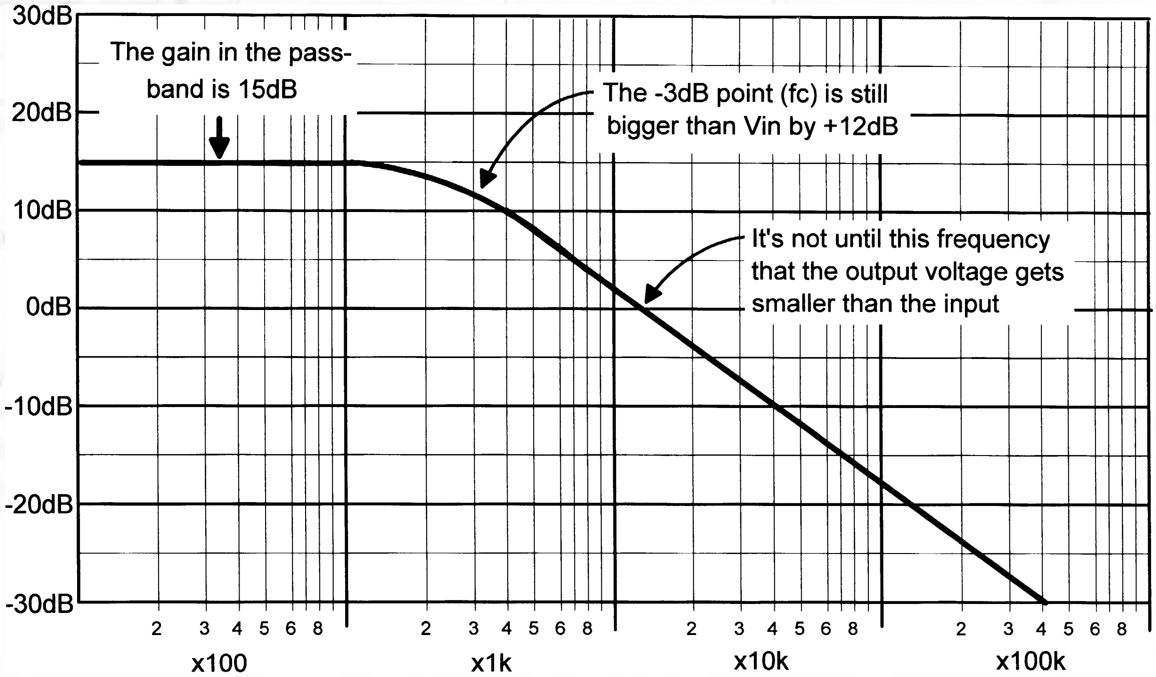


Figure 21 *The frequency response of an active filter with gain*

Skill practice 3

Practise determining the frequency performance of filters using an oscilloscope

This exercise is practise for the sorts of skills you may be required to perform in a practical test. Remember, in any practical tests you'll be working alone so make sure that you can perform all the steps. It should take you about 1½ hours to complete this exercise.

Equipment

- BWD minilab
- Loudspeaker crossover panel
- two BNC to banana-plug leads
- Banana leads

Remember:
Follow TAFE NSW WHS guidance at all times!

Work tasks

1. Read your WHS responsibilities at the top of the form below. Then conduct a WHS risk assessment and record your findings in the space provided.

Responsibilities of students under the Model WHS Act: s28

- Take reasonable care for your own health and safety by working safely at all times
- Take reasonable care to ensure that your acts or omissions don't put the health and safety of others at risk
- Follow all TAFE NSW WHS guidance and comply with all reasonable instructions from TAFE NSW staff to assist them in complying with the TAFE NSW WHS requirements
- In addition to the above, you must:
    - use and maintain machinery, tools and all other equipment properly and safely
    - ensure that your work area is free of hazards
    - notify a TAFE NSW staff member of actual or potential hazards
    - wear/use prescribed safety equipment
    - take notice of any safety signs and adhere to their instructions

Risks involved in this activity include:	**Control measures:**
Trip hazards (eg students bags) Objects dropped on feet (while equipment is taken to and from workbenches). Others: ______________ ______________ ______________	Move bags and other objects from walkways Plan lifting of equipment Other: ______________ ______________ ______________

My signature here indicates that I have read and understand my responsibilities under the Model WHS Act s28 (detailed above). I have also conducted a risk assessment before undertaking this activity and have identified measures to control these risks and have implemented them.

Signature: ______________ Date: ______________

2. Gather the equipment needed for this exercise.

3. Set the BWD Minilab's slider-switch for the power amp is set to the middle position.

4. Turn the Minilab off and connect the circuit of Figure 1.

   Note 1: Ensure that the black clips of the CRO leads connect to the panel's common ("C") sockets.

   Note 2: Ensure that the input to the Minilab's power amplifier is taken from the **0-10V** output of the Minilab's function generator.

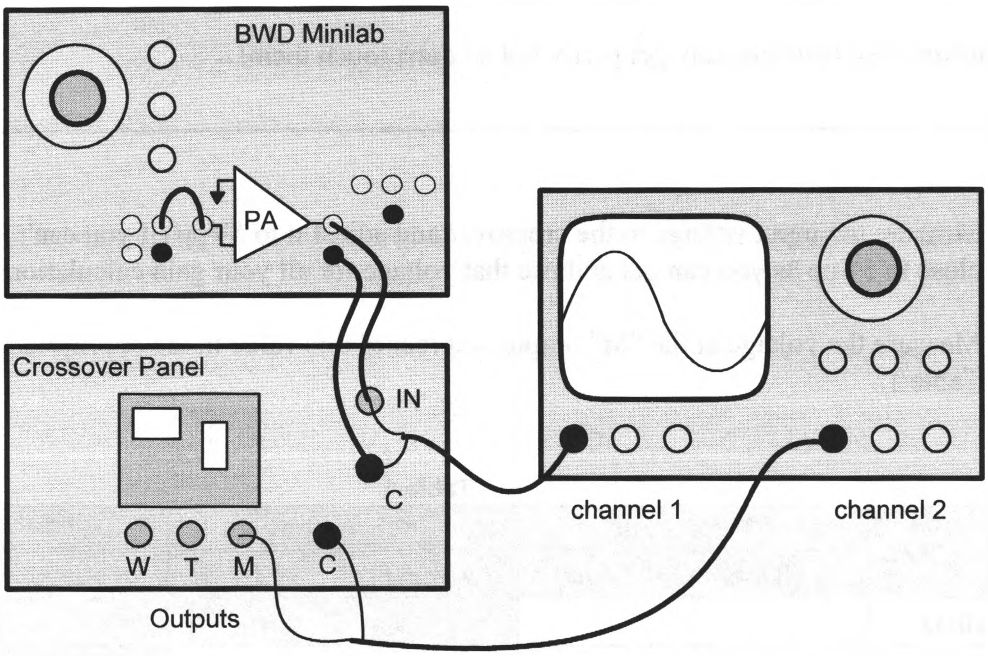


Figure 1

Note: It is very important that the loudspeaker crossover is driven from a low impedance source (to simulate the low impedance output of a power amplifier). Consequently, the Minilab must be used in this activity - and not the bench mounted function generator - so that the on-board power amplifier can be used. Take special care when setting up your equipment to make sure that this is wired correctly.

5. Adjust the controls on the Minilab's function generator for a 2kHz sinewave at the minimum amplitude.

6. Turn on the Minilab and adjust the function generator's amplitude control to produce the maximum peak-to-peak undistorted sinewave possible at the power amplifier's output (seen on channel 1 of the CRO).

7. Observe the "M" output of the crossover (seen on channel 2 of the CRO).

   Note: The signal should be about the same voltage as crossover's input voltage. If it's not, check your wiring.

Caution! The resistors may get pretty hot so don't touch them!

8. Measure the input voltage to the crossover and adjust it to 5Vpp (if you can't, set it for as close to 5Vpp as you can get and use that voltage for all your gain calculations).

9. Measure the voltage at the "M" output and record this value in the appropriate space on Table 1.

Table 1

Frequency	"M" output		"T" output		"W" output	
	Voltage	Av (dB)	Voltage	Av (dB)	Voltage	Av (dB)
20Hz						
40Hz						
100Hz						
200Hz						
400Hz						
1kHz						
2kHz						
4kHz						
8kHz						
10kHz						
12kHz						
14kHz						
18kHz						
22kHz						

10. Calculate the gain of the "M" output in decibels using the equation: $Av_{(dB)} = 20Log\frac{Vout}{Vin}$.

11. Repeat steps 9 and 10 for the "T" and "W" outputs.

12. Repeat steps 9 to 11 for all of the remaining frequencies listed in Table 1 (except for the shaded cells).

13. Plot the graph of frequency versus gain (in decibels) for each output using the log/linear graph paper on the next page.

Question 1
What type of filter is the "M" output of the crossover?

____

Question 2
What type of loudspeaker would you connect this output to?

____

Question 3
What type of filter is the "T" output of the crossover?

____

Question 4
What type of loudspeaker would you connect this output to?

____

Question 5
What type of filter is the "W" output of the crossover?

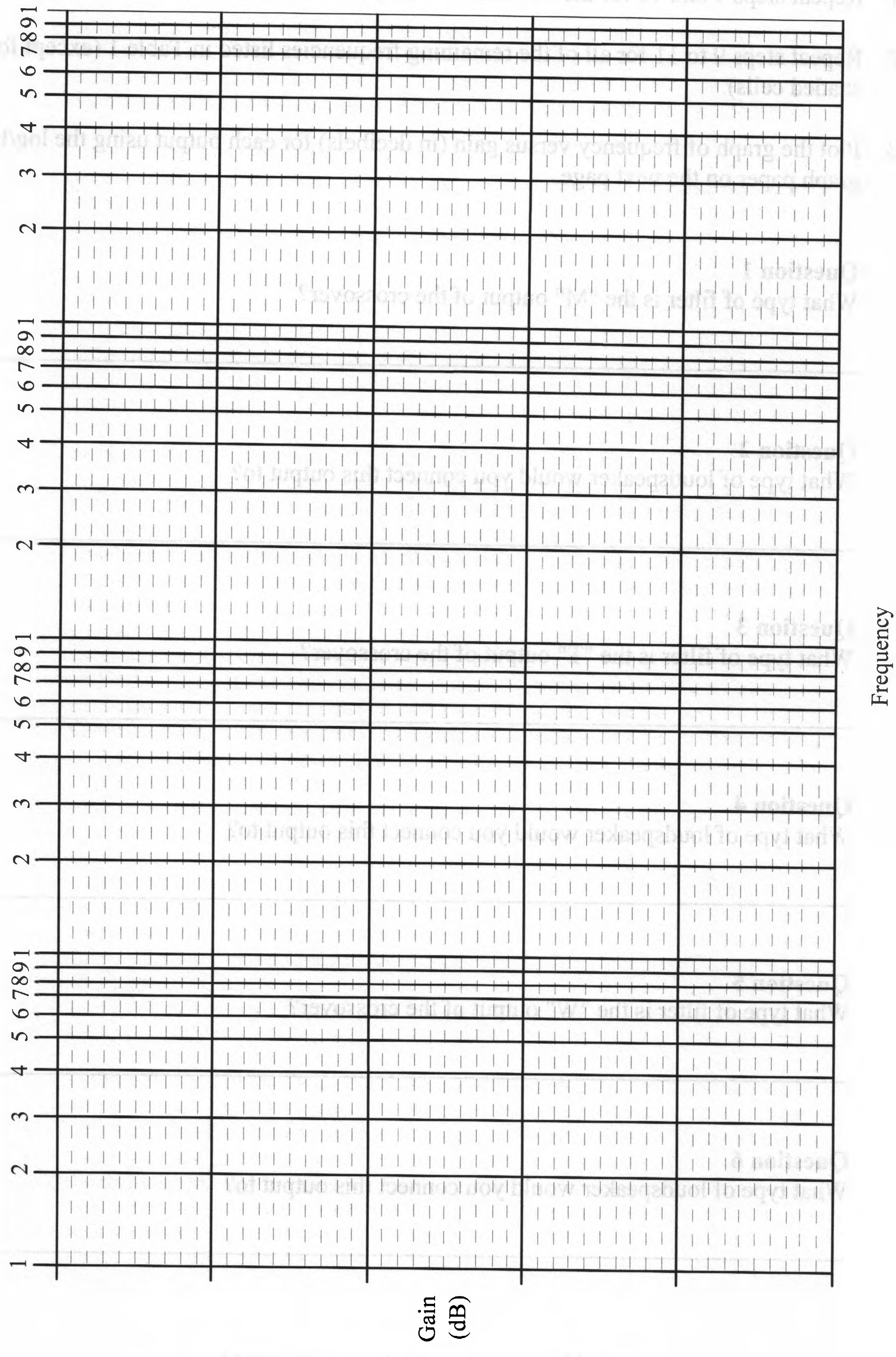
____

Question 6
What type of loudspeaker would you connect this output to?

____

More questions following the graph...

Note: Use a different colour pen for each output.



The teacher needs to check your work at this point...

Question 7
Find the approximate lower and upper frequency roll-off points (*f1* and *f2*) for the "M" output using the frequency response graph you have plotted.

Question 8
Find the approximate cut-off frequency (*fc*) for the "T" output using the frequency response graph you have plotted.

Question 9
Find the approximate cut-off frequency (*fc*) for the "W" output using the frequency response graph you have plotted.

Question 10
What is the insertion loss for each of the three filters?

Question 11
Determine the slope in dB/octave for both the lower and upper frequency roll-off of the "M" output using the graph you have plotted.

Question 12
Determine the slope in dB/octave for the lower and upper frequency roll-off of the "T" and "W" outputs.

The teacher needs to check your work at this point...

If time permits...

14. Set the frequency of the function generator's output signal to 2kHz.

15. Monitor the mid-range ("M") output using the CRO's channel 2 input.

16. Adjust the input and output signals on the CRO's screen so that they almost overlap each other.

17. Increase the frequency of the function generator's output signal while watching the CRO's screen. Describe what change takes place between the two signals. (Note: It isn't only the output voltage that changes!)

18. Return the frequency of the function generator to 2kHz.

19. Decrease the frequency of the function generator output signal while watching the CRO's screen. Describe what change takes place between the two signals.

The teacher needs to check your work at this point...

Review questions

Answer these questions to check your understanding of what you have learnt for this chapter. Doing this will also help to prepare you for the tests.

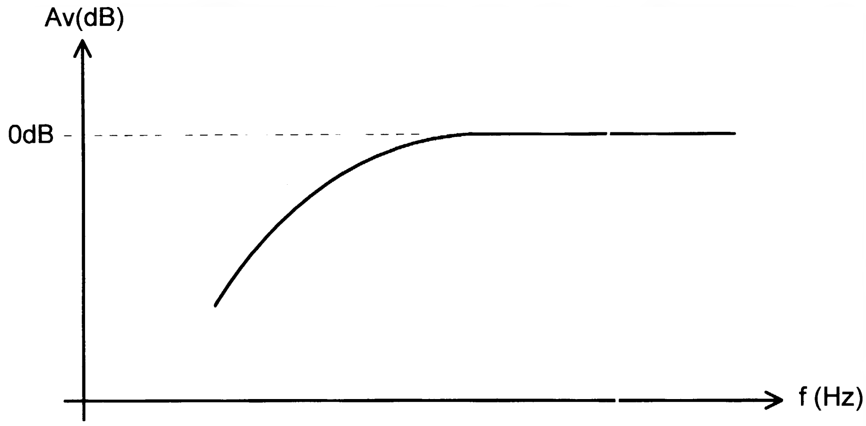


Figure 1

1. Show on the graph in Figure 1 what part of the response the following terms describe:

   (a) the pass-band

   (b) the cut-off frequency

   (c) the roll-off

2. According to the graph, the filter's output voltage is maximum at

   ☐ frequencies in the pass-band.

   ☐ the cut-off frequency.

   ☐ frequencies in the roll-off region.

3. The output voltage gets smaller in the roll-off region because the

   ☐ input signal gets smaller.

   ☐ filter's gain goes down

   ☐ input is inverted.

   ☐ filter has an insertion loss.

Questions 4 to 9 refer to the graph in Figure 2

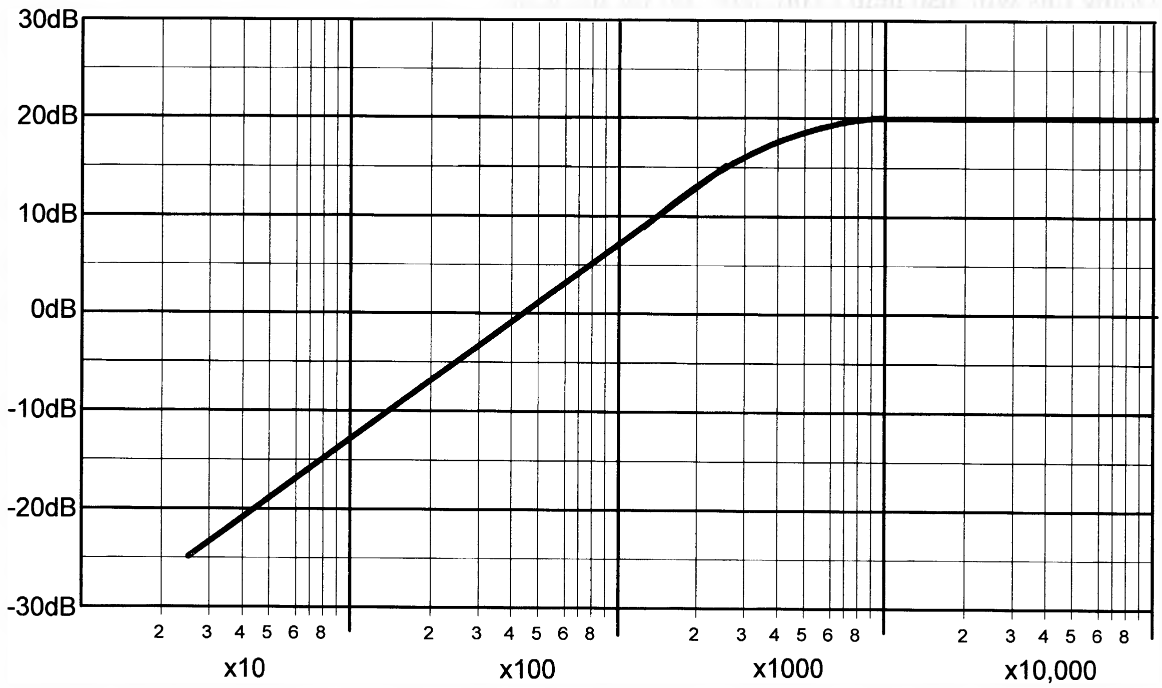


Figure 2

4. What type of filter does the graph show?

- ☐ Active low-pass filter
- ☐ Passive low-pass filter
- ☐ Active high-pass filter
- ☐ Passive high-pass filter

5. What is the slope of the roll-off?

- ☐ 10dB/decade
- ☐ 20dB/decade
- ☐ 30dB/decade
- ☐ 40dB/decade

6. What is the pass-band gain?

____

7. What is the cut-off frequency?

8. At what frequency (approximately) is the output voltage the same as the input voltage?

9. What is the filter's order?

Questions 10 to 12 refer to Figure 3

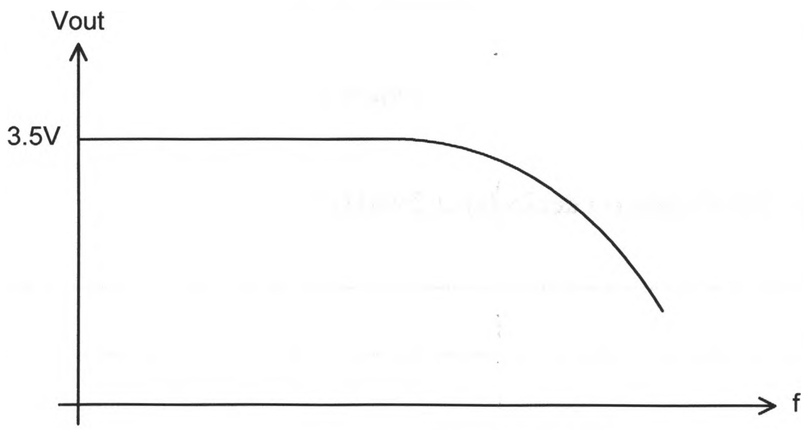


Figure 3

10. What type of filter is represented by the frequency response shown?

11. Calculate the filter's pass-band gain in decibels if its input voltage is 9V.

12. Calculate the filter's output voltage at the cut-off frequency.

Questions 13 to 18 refer to the first order filter in Figure 4

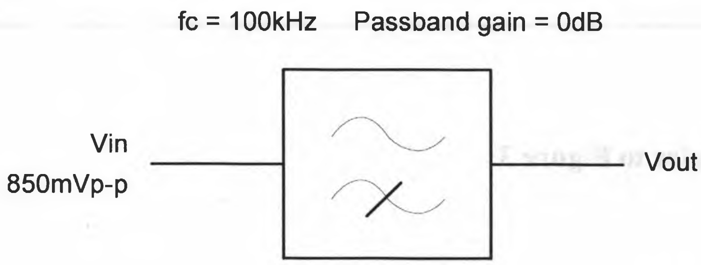


Figure 4

13. What is the filter's gain (in decibels) at 250kHz?

14. What is the output voltage at 250kHz?

15. Calculate the filter's output voltage at the cut-off frequency (*fc*).

16. What is the filter's gain at 5kHz if its gain at 50kHz is -12dB?

17. Calculate the filter's output voltage at 5kHz.

Questions 18 to 20 refer to Figure 5

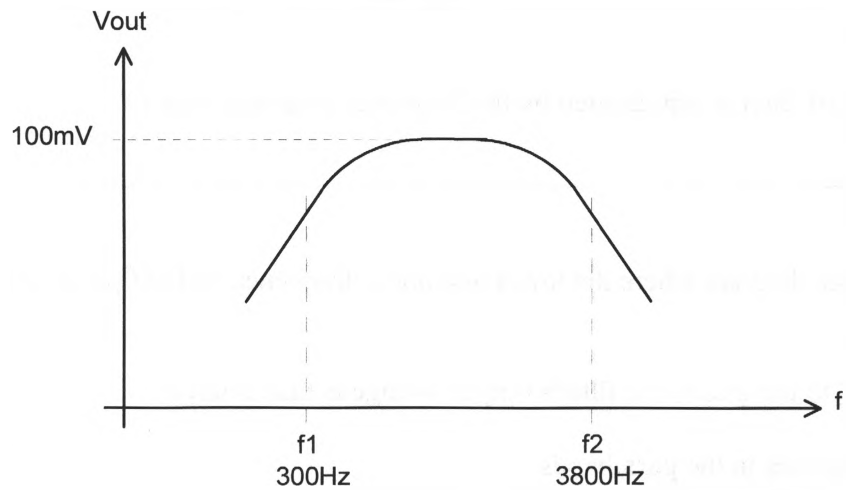


Figure 5

18. What type of filter is represented by the frequency response shown?

19. Calculate the filter's bandwidth.

20. Calculate the filter's centre frequency from the *f1* and *f2*.

Questions 21 to 25 refer to Figure 6

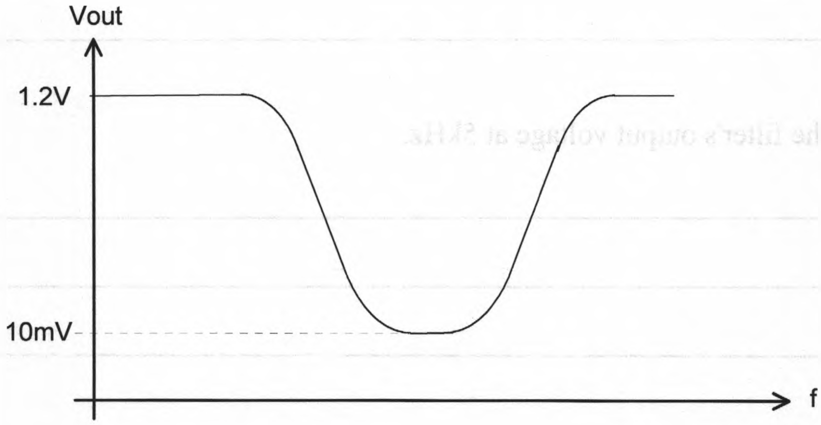


Figure 6

21. What type of filter is represented by the frequency response shown?

____

22. Show on the diagram where the lower and upper frequency roll-off points (*f1* & *f2*) occur.

23. According to the graph, the filter's output voltage is maximum at

- ☐ frequencies in the pass-bands.
- ☐ the cut-off frequency.
- ☐ frequencies outside the stop-band.

24. Calculate the filter's output voltage at *f1* and *f2*.

____

____

25. Calculate the filter's gain in the stop-band if the input voltage is 1.2V.

____

____

Section 4 Low-pass and high-pass filter circuits

Purpose

To develop your ability to identify passive filters in electronic systems and determine whether they are functioning correctly.

Objectives

At the end of this section you should be able to:

- Identify passive RC and RL low-pass and high-pass filter circuits
- Specify the slope of the roll-off of passive first order RC and RL low-pass and high-pass filter circuits
- State the relative phase relationship between input and output voltages at a given frequency for passive RC and RL low-pass and high-pass filter circuits
- Specify the phase relationship between input and output voltages at the cut-off frequency for passive RC and RL low-pass and high-pass filter circuits
- Calculate the cut-off frequency of passive first order RC and RL low-pass and high-pass filter circuits
- Perform frequency response tests on simple RC and RL low-pass and high-pass filters

Introduction

By now, you'll appreciate that there are many types of filters. There are low-pass filters, high-pass filters, band-pass filters and band-stop filters. Filters can be first order, second order, third order filters and so on. They can be active or passive. They can be designed with different cut-off frequencies, different pass-bands, stop-bands and sometimes they have an insertion loss. In this section and in Section 6 specific filter circuits made from capacitors and/or inductors are explained.

Low-pass filter circuits

The passive 1st order RC low-pass filter

One of the simplest low-pass filter circuits can be made using a single resistor and a capacitor. The circuit is shown in Figure 1 below.

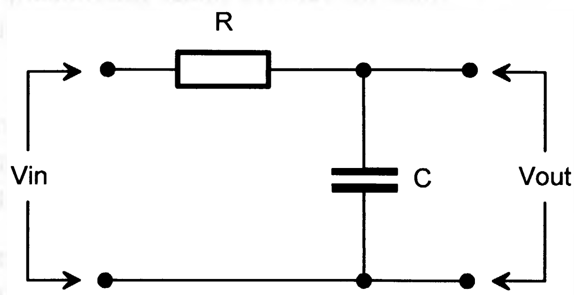


Figure 1 *A passive first order RC low-pass filter*

The circuit is a passive filter because it is only made from passive components (that is, there are no amplifying devices). It is a first order filter because it has only one reactive component. This means that it has a roll-off of 6dB/octave (or 20dB/decade).

Operation

The principle of operation of this circuit is basically a voltage divider. When an input signal (*Vin*) is connected to the circuit, its voltage is divided between the two components. As the circuit's output terminals are connected across the capacitor, the potential difference across the capacitor (V_C) and the output voltage (*Vout*) are the same as each other.

When the input signal is a relatively low frequency the capacitor's reactance is very large relative to the resistor's value. That being the case, almost all of the input voltage is dropped across the capacitor leaving almost nothing for the resistor. So at relatively low frequencies the output voltage is the same as the input voltage.

When the input signal is an intermediate frequency (that is, neither relatively low nor relatively high) the capacitor's reactance is lower and doesn't seem as large relative to the resistor's value. This means that more of the input voltage is dropped across the resistor and less dropped across the capacitor. So at intermediate frequencies the output voltage is smaller than the input voltage and gets smaller as the input signal's frequency increases.

When the input signal is a relatively high frequency the capacitor's reactance is very small relative to the resistor's value. In which case, most of the input voltage is dropped across the resistor and the capacitor has almost none. So at relatively high frequencies the output voltage is very small and approaches zero volts.

In summary, the circuit allows relatively low frequency input signals to pass through to the output mostly unaffected but attenuates middle range frequencies by a reasonable amount and relatively high frequencies by a lot. This is the operation of a practical low-pass filter.

This can be proven mathematically using a circuit with component values. The calculations here are straight from work you have done in the first part of this semester so there's nothing new here.

Figure 2 shows the RC low-pass filter made from a 1kΩ resistor, a 100nF capacitor with a 10Vp-p sinewave on the input.

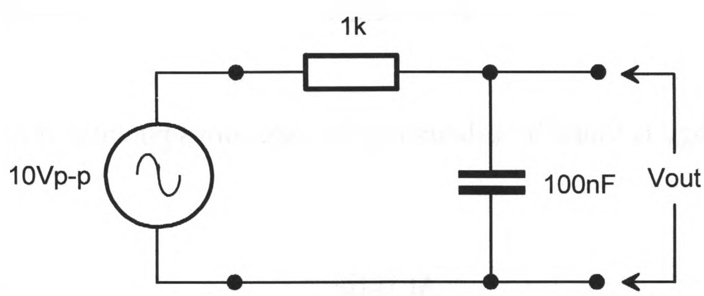


Figure 2

The following analysis shows the output voltage calculated at three frequencies: 100Hz, 1kHz and 100kHz.

First, the capacitive reactance is found using the equation: $X_C = \frac{1}{2\pi fC}$.

At 100Hz	At 1kHz	At 100kHz
$X_C = \frac{1}{2\pi \times 100\text{Hz} \times 100\text{nF}}$	$X_C = \frac{1}{2\pi \times 1\text{kHz} \times 100\text{nF}}$	$X_C = \frac{1}{2\pi \times 100\text{kHz} \times 100\text{nF}}$
$X_C = 15.9\text{k}\Omega$	$X_C = 1.59\text{k}\Omega$	$X_C = 15.9\Omega$

Next, the circuit's impedance is found using: $Z = \sqrt{R^2 + X_C^2}$.

At 100Hz	At 1kHz	At 100kHz
$Z = \sqrt{1k^2 + 15.9k^2}$	$Z = \sqrt{1k^2 + 1.59k^2}$	$Z = \sqrt{1k^2 + 15.9^2}$
$Z = 15.95k\Omega$	$Z = 1.88k\Omega$	$z = 1k\Omega$

To find the output voltage the circuit current must first be found using: $I = \frac{E}{Z}$.

At 100Hz	At 1kHz	At 100kHz
$I = \frac{10V}{15.95k}$	$I = \frac{10V}{1.88k}$	$I = \frac{10V}{1k}$
$I = 627\mu A$	$I = 5.32mA$	$I = 10mA$

Finally, the output voltage is found by calculating the capacitor's potential difference using: $Vout = I \times X_C$.

At 100Hz	At 1kHz	At 100kHz
$Vout = 627\mu A \times 15.9k\Omega$	$Vout = 5.32mA \times 1.59k\Omega$	$Vout = 10mA \times 15.9\Omega$
$Vout = 9.97V$	$Vout = 8.46V$	$Vout = 159mV$

Plotting these results on a log/linear graph paper shows that the circuit is behaving as a low pass filter (see Figure 3).

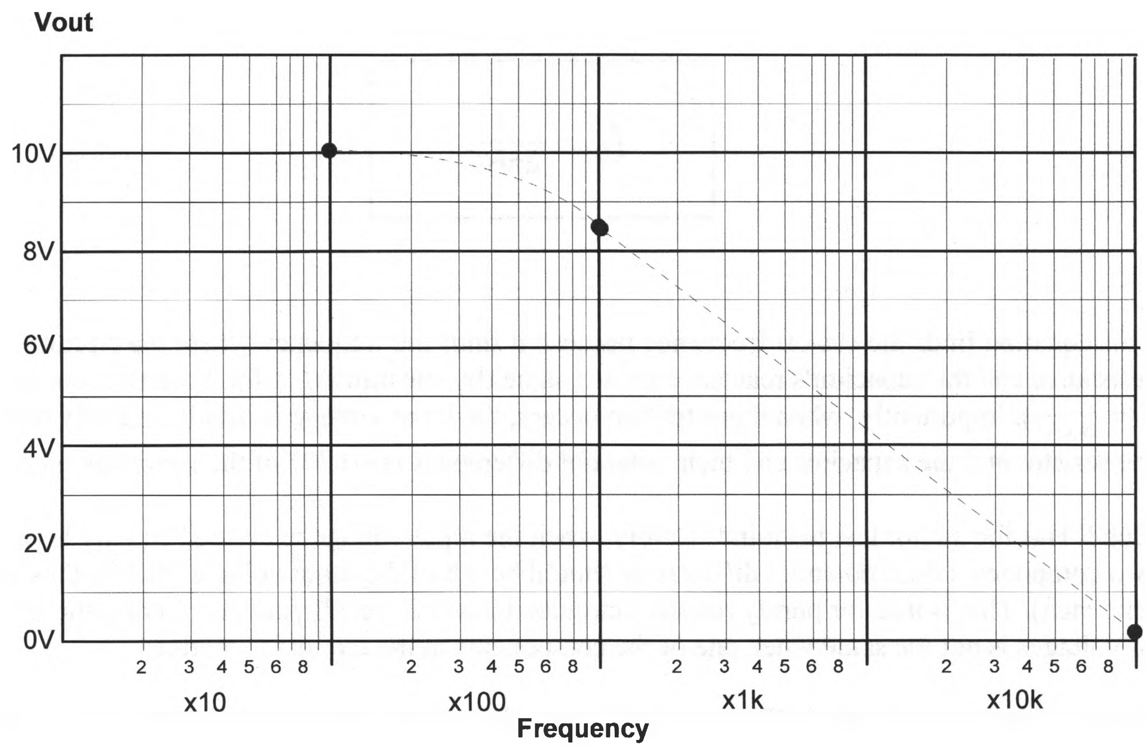


Figure 3 *The graph of the filter's output voltage calculated at three frequencies*

Calculating the cut-off frequency

Recall that the cut-off frequency is the point where the filter's output voltage and (hence its gain) drops to 0.707 of the maximum. To calculate the cut-off frequency (*fc*) we use the equation:

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

This equation finds the cut-off frequency because it finds the frequency where the circuit's resistance and the capacitor's reactance are the same (by substituting R for X_C in the equation $f_c = \frac{1}{2\pi X_C C}$). Importantly, when this situation occurs, the input voltage is divided equally between the resistor and the capacitor and their potential differences are 0.707 of the input voltage.

Huh?! Has the author lost his mind? Surely, when the input voltage is divided equally between two components their potential differences should be 0.5 of the input voltage (that is, they get half each). This is true for purely resistive circuits. However, recall your AC theory, the division of voltages is not the same when one of the components in the circuit is reactive.

To prove this, consider the LPF circuit in Figure 4. Its cut-off frequency has been calculated using the equation above and is shown.

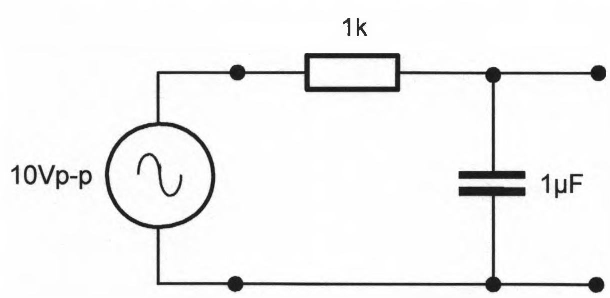


Figure 4

$$fc = \frac{1}{2\pi RC}$$

$$fc = \frac{1}{2\pi \times 1k\Omega \times 1\mu F}$$

$$fc = 159Hz$$

Now let's use this value to work backwards to calculate the potential difference across the two components.

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi \times 159Hz \times 1\mu F}$$

$$X_C = 1k\Omega$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{1k\Omega^2 + 1k\Omega^2}$$

$$Z = 1414.2\Omega$$

$$I = \tfrac{E}{Z}$$

$$I = \frac{10Vp\text{-}p}{1414.2\Omega}$$

$$I = 7.07mAp\text{-}p$$

$$V_C = I \times X_C$$

$$V_C = 7.07mAp\text{-}p \times 1k\Omega$$

$$V_C = 7.07Vp\text{-}p$$

$$V_R = I \times R$$

$$V_R = 7.07mAp\text{-}p \times 1k\Omega$$

$$V_R = 7.07Vp\text{-}p$$

See! At 159Hz the potential difference across the two components is 0.707 of the input voltage. And, as *Vout* is the same as the potential difference across the capacitor, 159Hz must be the cut-off frequency.

Practise using the equation by finding the cut-off frequency of the circuit below.

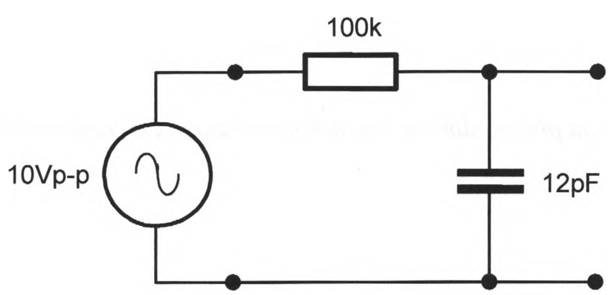


Figure 5

fc = ____

____

Output voltage phase shift

A signal's amplitude is not the only thing that an RC low-pass filter can change. A phase shift between the input and output signals can also be introduced. Sometimes this phase shift is desirable (and is the reason the filter is being used) but usually it's undesirable and not wanted.

The amount of phase shift will depend on the input signal's frequency and the component values. It can range from no phase shift for most of the frequencies in the pass band to the output lagging the input by nearly 90° at very high frequencies.

A graph of frequency versus phase shift is shown in Figure 6. To make it more meaningful, it is shown coincident with the filter's gain.

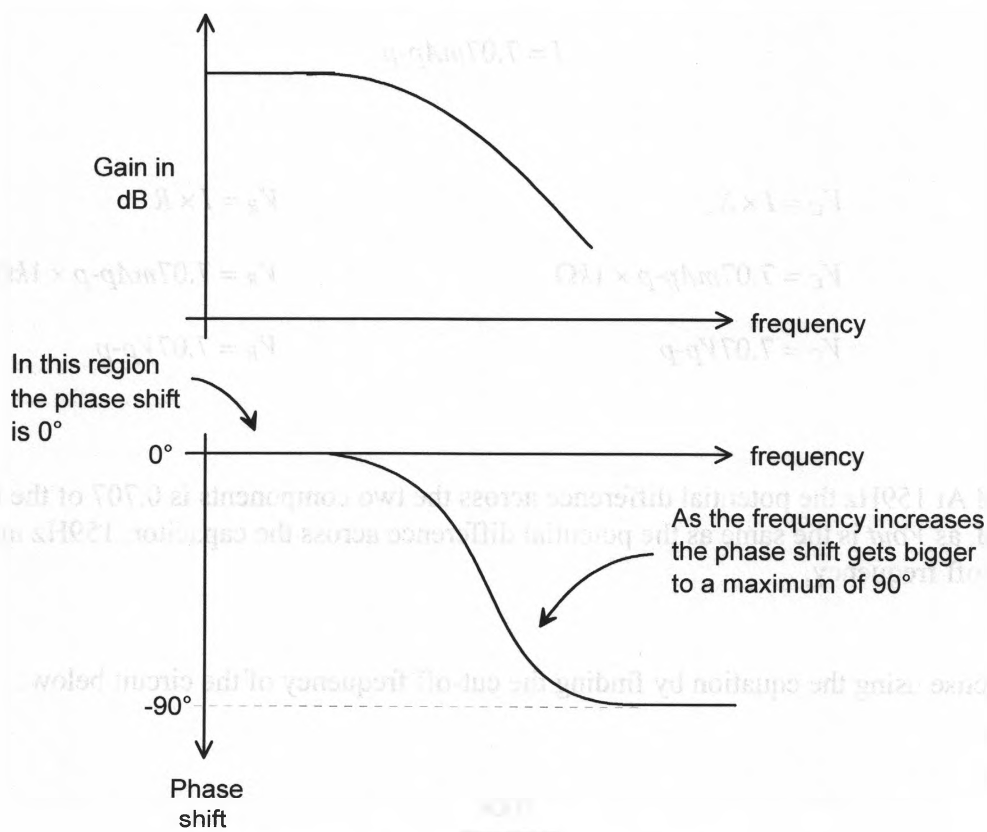


Figure 6 *Graph of frequency versus phase shift for the output voltage of a passive RC low-pass filter*

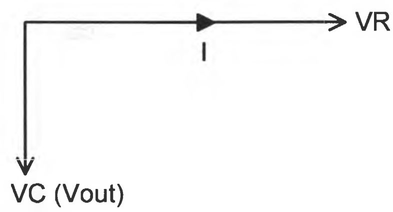
A way to help understand why the output voltage of an RC low-pass filter lags the input voltage at frequencies in the filter's roll-off involves using a phasor diagram.

Recall that the current in a series RC circuit and the potential difference across the resistor are always in phase. This is shown in the phasor diagram in Figure 7a.

7a

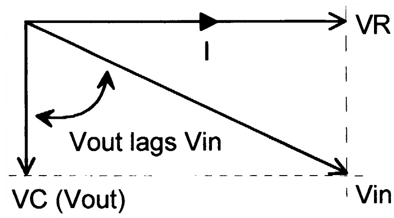
Recall also that the potential difference across the capacitor in a series RC circuit always lags the circuit current by 90°. This is shown in Figure 7b.

7b



As the supply voltage is the vectorial sum of the potential differences across the resistor and the capacitor, it must fall between V_R and V_C. When this is shown in the phasor diagram (see Figure 7c) it can be seen that *Vout* lags *Vin*.

7c



The output phase shift at the cut-off frequency

At the cut-off frequency the phase shift between the input and output signals is exactly -45°. That is, at *fc Vout* lags *Vin* by 45° (see Figure 8 below).

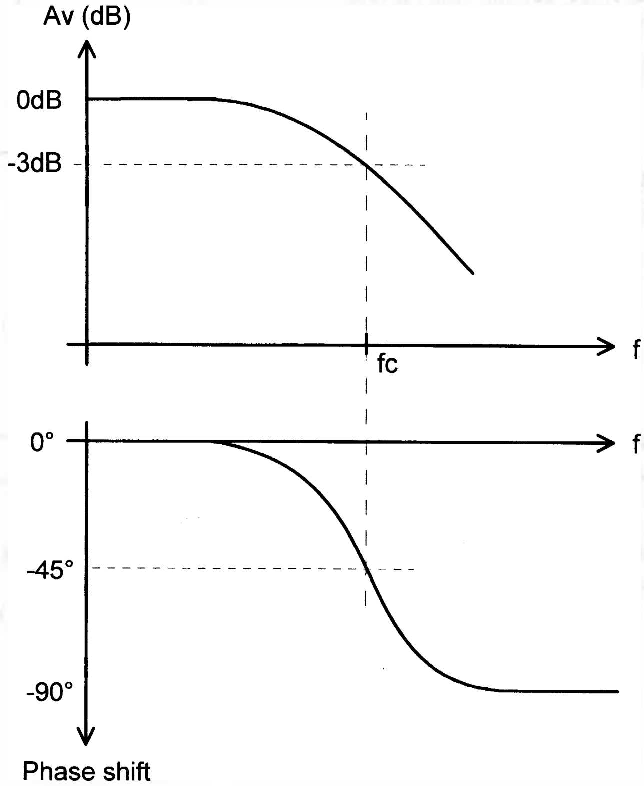


Figure 8 *Vout lags Vin by 45° at the cut-off frequency*

The passive 1st order RL low-pass filter

Another simple low-pass filter can be made from just a resistor and an inductor. The circuit is shown in Figure 9 below.

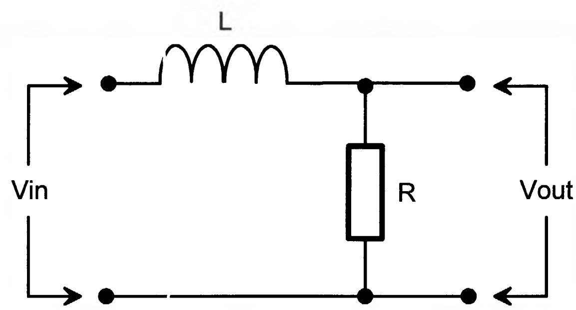


Figure 9 *A passive first order RL low-pass filter*

The circuit is passive because it is only made from passive components. It's a first order filter because it has only one reactive element, the inductor.

Operation

Like the RC low-pass filter, we can think of this circuit as a voltage divider. When an input signal (Vin) is connected to the circuit its voltage is divided between the two components. As the circuit's output terminals are connected across the resistor, the potential difference across the resistor (V_R) and the output voltage ($Vout$) are the same as each other.

When the input signal is a relatively low frequency the inductor's reactance is very small relative to the resistor's value. That being the case, almost all of the input voltage is dropped across the resistor leaving almost nothing for the inductor. So at relatively low frequencies the output voltage is the same as the input voltage.

When the input signal is an intermediate frequency the inductor's reactance is bigger and doesn't seem as small relative to the resistor's value. This means that more of the input voltage is dropped across the inductor and less dropped across the resistor. So at intermediate frequencies the output voltage is smaller than the input voltage and gets smaller as the input signal's frequency increases.

When the input signal is a relatively high frequency the inductor's reactance is very large relative to the resistor's value. In which case, most of the input voltage is dropped across the inductor and the resistor has almost none. So at relatively high frequencies the output voltage is very small and approaches zero volts.

In summary, the circuit allows relatively low frequency signals on the input to pass through to the output mostly unaffected but attenuates middle range frequencies by a reasonable amount and relatively high frequencies by a lot. This is the operation of a practical low-pass filter.

Calculating the cut-off frequency
Calculating the cut-off frequency (*fc*) of the RL low-pass filter is simple. As a starting point we use the equation $X_L = 2\pi fL$ and transpose the equation to make *f* the subject. This gives: $f = \frac{X_L}{2\pi L}$.

As with the RC low-pass filter, the inductive reactance at the cut-off frequency is equal to the resistor's value. So, if we substitute *R* for X_L, the equation becomes:

$$\boxed{f_C = \frac{R}{2\pi L}}$$

Practise using the equation by finding the cut-off frequency of the circuit below.

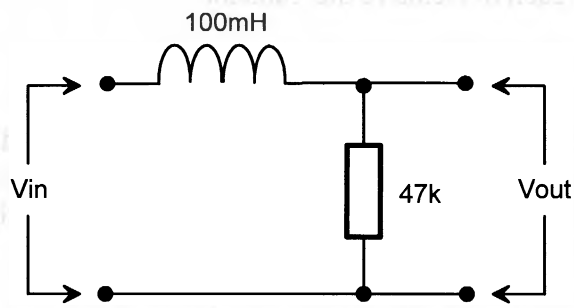


Figure 10

fc =

Output voltage phase shift
As with RC low-pass filter circuits, the input and output signals of RL low-pass filters are not in phase at some frequencies. The amount of phase shift depends on the input signal's frequency and component values. It can range from no phase shift for most of the frequencies in the pass band to the output lagging the input by nearly 90° at very high frequencies.

A graph of frequency versus phase shift is shown in Figure 11 (on the next page). The phase shift between the input and output signals is exactly -45° at the cut-off frequency.

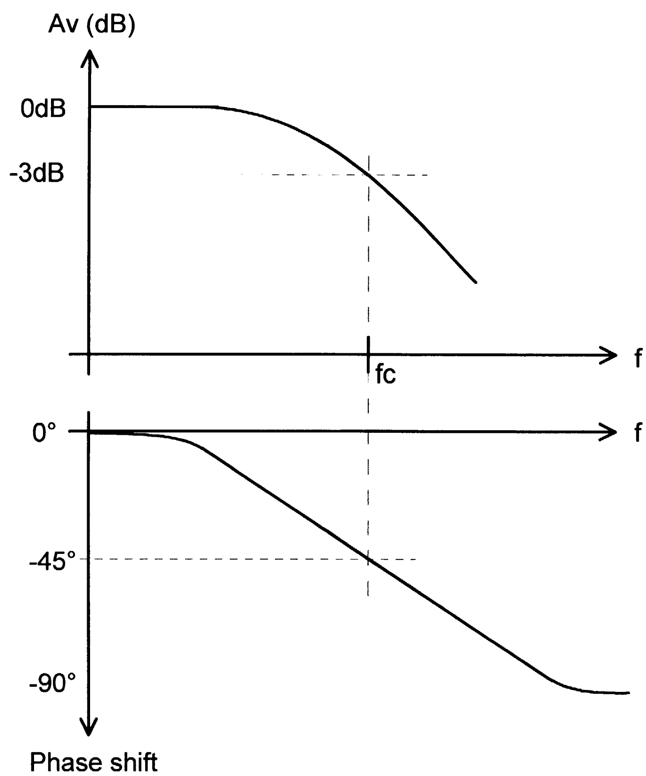
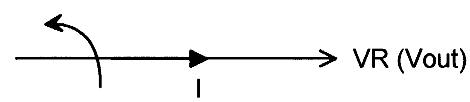


Figure 11 *Graph of frequency versus phase shift for the output voltage of a passive RL low-pass filter*

As voltage leads current in an inductor, it is tempting to think that the output voltage should lead the input voltage. However, this is not the case. A way to appreciate why the output voltage lags the input voltage instead of leading it is to use a phasor diagram.

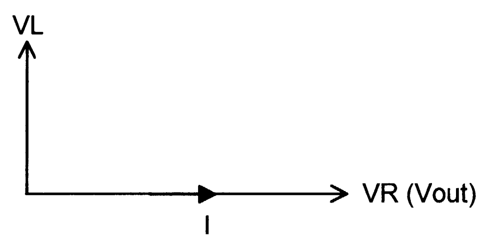
The circuit current and the potential difference across the resistor (which is also the output voltage) are in phase. This is shown in Figure 12a.

12a



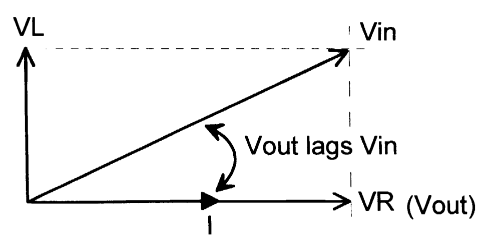
The current in the inductor and the resistor are the same because they are in series. However, the potential difference across the inductor leads the current through it by 90°. This is shown in Figure 12b.

12b



As the supply voltage is the vectorial sum of the potential differences across the resistor and inductor, it must fall between V_R and V_L (see Figure 12c). In this way, *Vout* can be seen to be lagging *Vin*.

12c



High-pass filter circuits

The passive 1st order RC high-pass filter

One of the simplest high-pass filter circuits can be made using just a resistor and a capacitor. The circuit is shown in Figure 13 below.

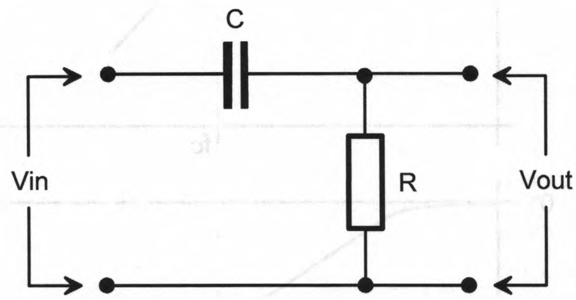


Figure 13 *A passive first order RC high-pass filter*

The circuit is passive because it is only made from passive components. It's a first order filter because it has only one reactive element, the capacitor.

Operation

When the input signal is a relatively low frequency the capacitor's reactance is very large relative to the resistor's value. That being the case, almost all of the input voltage is dropped across the capacitor leaving almost nothing for the resistor. So at relatively low frequencies the output voltage is very small and approaches zero volts.

When the input signal is an intermediate frequency the capacitor's reactance is smaller and isn't as big relative to the resistor's value. This means that more of the input voltage is dropped across the resistor and less dropped across the capacitor. So at intermediate frequencies the output voltage is no-longer zero volts and gets bigger as the input signal's frequency increases.

When the input signal is a relatively high frequency the capacitor's reactance is very small relative to the resistor's value. In which case, most of the input voltage is dropped across the resistor and the capacitor has almost none. So at relatively high frequencies the output voltage is the same as the input voltage.

In summary, the circuit allows relatively high frequency signals on the input to pass through to the output mostly unaffected but attenuate middle range frequencies by a reasonable amount and relatively low frequencies by a lot. This is the operation of a practical high-pass filter.

Calculating the cut-off frequency

At the cut-off frequency the capacitor's reactance is equal to the resistor so *fc* can be found using:

$$\boxed{f_C = \frac{1}{2\pi RC}}$$

Output voltage phase shift

As with RC low-pass filter circuits, the input and output signals of RC high-pass filters are not in phase at some frequencies. The amount of phase shift depends on the input signal's frequency and component values. It can range from no phase shift for most of the frequencies in the pass band to the output leading the input by nearly 90° at low frequencies.

A graph of frequency versus phase shift is shown in Figure 14. The phase shift between the input and output signals is exactly +45° at the cut-off frequency.

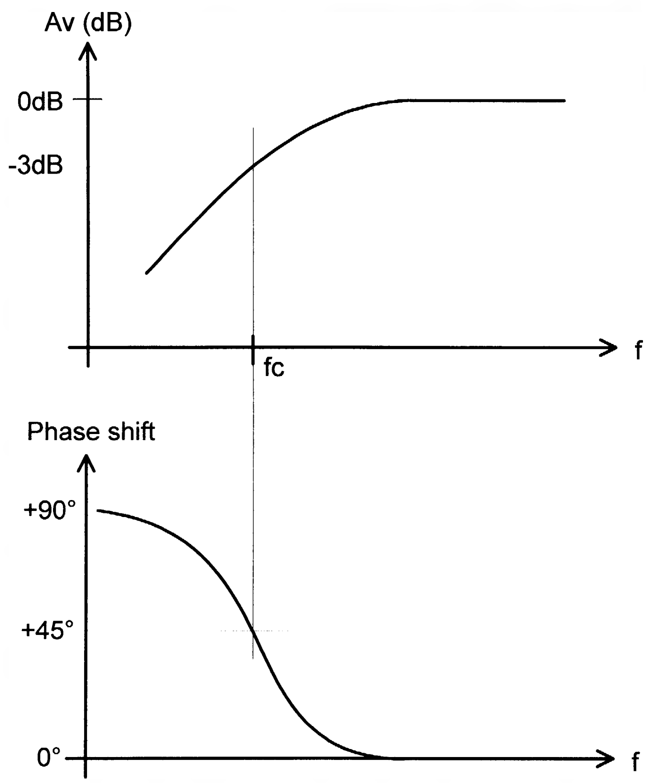


Figure 14 *Graph of frequency versus phase shift for the output voltage of a passive RC high-pass filter*

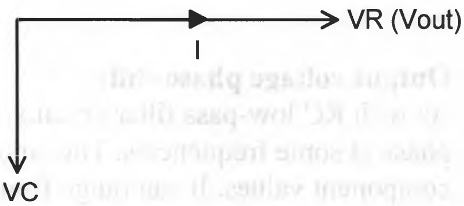
As voltage lags current in a capacitor, it is tempting to think that the output voltage should lag the input voltage, after all it did in an RC low-pass filter. However, this is not the case. A way to appreciate why the output voltage leads the input voltage instead of lagging it is to use a phasor diagram.

The circuit current and the potential difference across the resistor (which is also the output voltage) are in phase. This is shown in Figure 15a.

15a

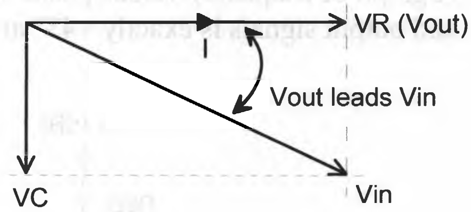
The potential difference across the capacitor in a series RC circuit always lags the circuit current by 90°. This is shown in Figure 15b.

15b



As the supply voltage is the vectorial sum of the potential differences across the resistor and the capacitor, it must fall between V_R and V_C (see Figure 15c). In this way, *Vout* can be seen to be leading *Vin*.

15c



The passive 1st order RL high-pass filter

Another simple high-pass filter can be made from just a resistor and an inductor. The circuit is shown in Figure 16 below.

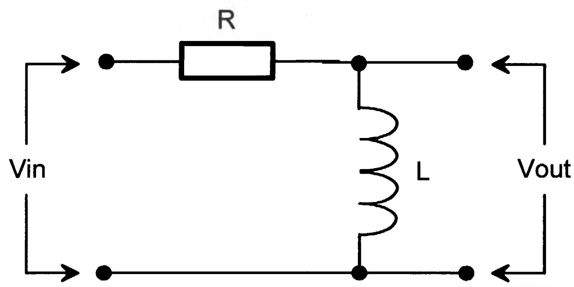


Figure 16 *A passive first order RL high-pass filter*

The circuit is passive because it is only made from passive components. It's a first order filter because it has only one reactive element, the inductor.

Operation

When the input signal is a relatively low frequency the inductor's reactance is very small relative to the resistor's value. That being the case, almost all of the input voltage is dropped across the resistor leaving almost nothing for the inductor. So at relatively low frequencies the output voltage is very small and approaches zero volts.

When the input signal is an intermediate frequency the inductor's reactance is bigger and isn't as small relative to the resistor's value. This means that more of the input voltage is dropped across the inductor and less dropped across the resistor. So at intermediate frequencies the output voltage is no-longer zero volts and gets bigger as the input signal's frequency increases.

When the input signal is a relatively high frequency the inductor's reactance is very large relative to the resistor's value. In which case, most of the input voltage is dropped across the inductor and the resistor has almost none. So at relatively high frequencies the output voltage is the same as the input voltage.

In summary, the circuit allows relatively high frequency signals on the input to pass through to the output mostly unaffected but attenuate middle range frequencies by a reasonable amount and relatively low frequencies by a lot. This is the operation of a practical high-pass filter.

Calculating the cut-off frequency
At the cut-off frequency the inductor's reactance is equal to the resistor so *fc* can be found using:

$$\boxed{f_C = \frac{R}{2\pi L}}$$

Output voltage phase shift
As with RC high-pass filter circuits, the input and output signals of RL high-pass filters are not in phase at some frequencies. The amount of phase shift will depend on the input signal's frequency and component values. It can range from no phase shift for most of the frequencies in the pass band to the output leading the input by nearly 90° at low frequencies.

A graph of frequency versus phase shift is shown in Figure 17 below. The phase shift between the input and output signals is exactly +45° at the cut-off frequency.

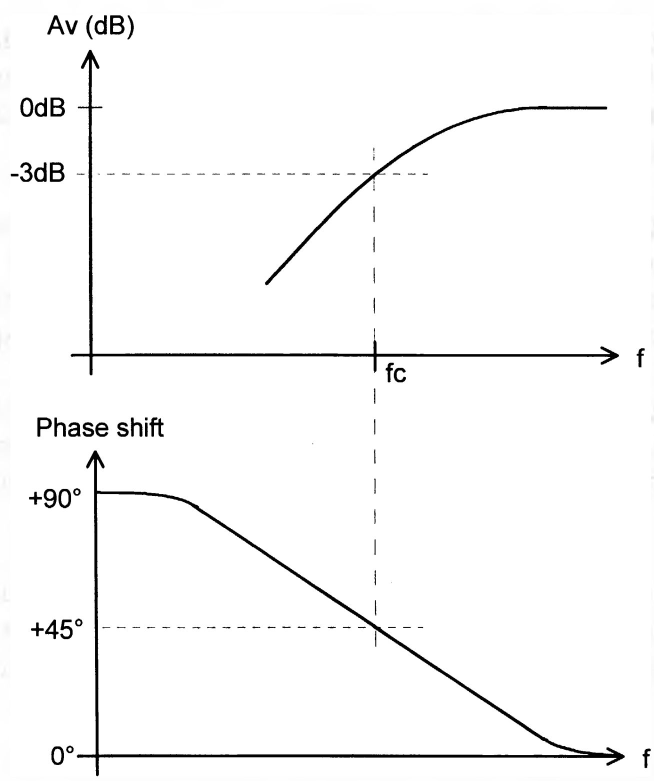


Figure 17 *Graph of frequency versus phase shift for the output voltage of a passive RL high-pass filter*

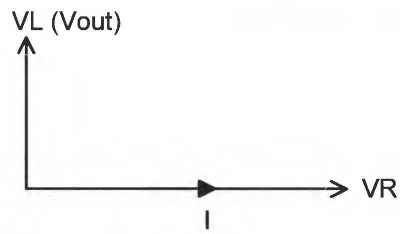
A way of appreciating why the output voltage can lead the input voltage is to use a phasor diagram.

The potential difference across the resistor and the current through it are in phase. This is shown in Figure 18a.

18a

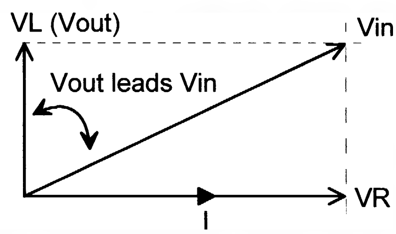
The current in the inductor and the resistor are the same because they are in series. However, the voltage across the inductor (which is the output voltage) leads the current through it by 90°. This is shown in Figure 18b.

18b



As the supply voltage is the vectorial sum of the potential differences across the resistor and inductor, it must fall between V_R and V_L (see Figure 18c). In this way, *Vout* can be seen to be leading *Vin*.

18c



Recognising the difference between RC and RL low-pass and high-pass filters

At this point, you may be thinking that you need a lot of theory to tell whether an RC or RL combination is a low-pass or a high pass filter. There is a simpler way of working out which filter is which that you may prefer to use. There are two steps to the analysis:

Step 1: Determine the output voltage at low frequencies

- Assume the input is a DC voltage
- Replace capacitors with open-circuits and inductors with short-circuits
- Determine whether the output voltage is 0V or the same as the input.

Step 2: Determine the output voltage at high frequencies

- Assume the input is an extremely high frequency AC signal
- Replace capacitors with short-circuits and inductors with open-circuits
- Determine whether the output voltage is 0V or the same as the input.

If the output voltage is maximum at low frequencies and minimum at high frequencies then the circuit must be a low-pass filter. If the output voltage is minimum at low frequencies and maximum at high frequencies then the circuit must be a high-pass filter.

Let's try the procedure out on a circuit:

Figure 19a is an RL circuit that we want to identify as being either a low-pass or a high-pass filter.

19a

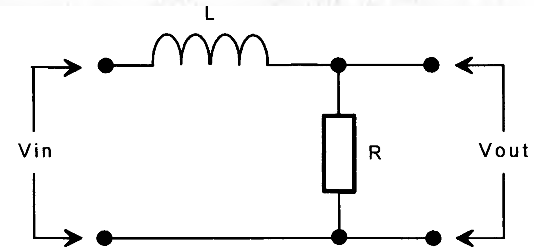


Figure 19b shows a DC voltage connected to the input and the inductor replaced with a short. From this diagram we can see that the input is directly connected to the output. Therefore, the output voltage is the same as the input voltage.

19b

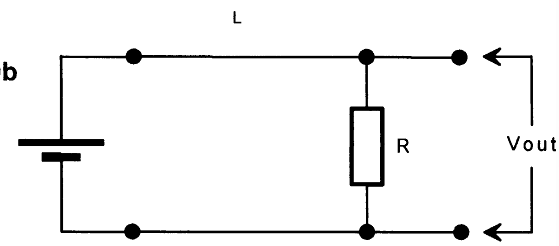
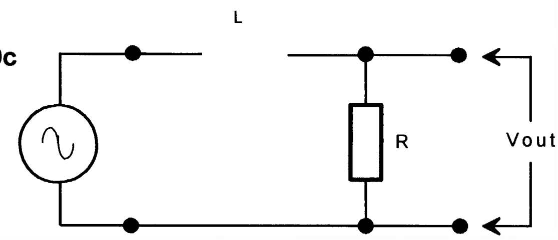


Figure 19c shows a high frequency AC signal connected to the input and the inductor replaced with an open-circuit. From this diagram we can see that there is no connection from input to output. Therefore, the output voltage is 0V.

19c



At low frequencies, the output voltage of the circuit in Figure 19 is the same as the input voltage and at high frequencies it is 0V. Therefore, this must be a low-pass filter.

Let's try the procedure on another circuit:

Figure 20a is an RC circuit that we want to identify as being either a low-pass or a high-pass filter.

20a

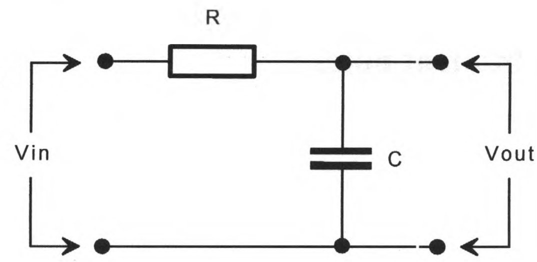


Figure 20b shows a DC voltage connected to the input and the capacitor replaced with an open-circuit. From this diagram we can see that, without the capacitor, there is no voltage divider and the input is directly connected to the output. Therefore, the output voltage is the same as the input voltage.

20b

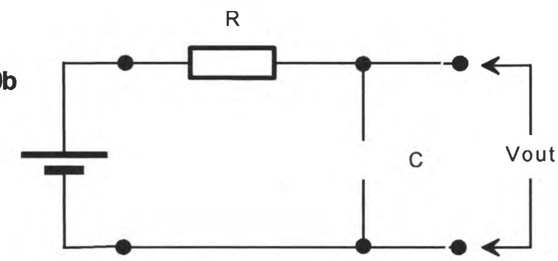
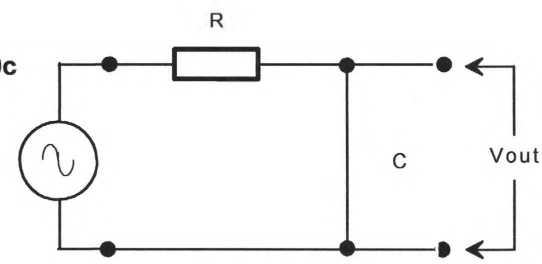


Figure 20c shows a high frequency AC signal connected to the input and the capacitor replaced with a short. From this diagram we can see that the output has a short circuit connected directly across it so the output voltage is 0V.

20c



At low frequencies, the output voltage of the circuit in Figure 20 is the same as the input voltage and at high frequencies it is 0V. Therefore, this must also be a low-pass filter.

Student notes

Skill practice 4

Passive RC low-pass and high-pass filters

This exercise is practise for the sorts of skills you may be required to perform in a practical test. Remember, in any practical tests you'll be working alone so make sure that you can perform all the steps. It should take you about 1¼ hours to complete this exercise.

Equipment

- Interface panel
- 10kΩ ¼W resistor
- 2.2nF capacitor
- three BNC to banana-plug leads
- Banana leads

Remember:

Follow TAFE NSW WHS guidance at all times!

Work tasks

1. Read your WHS responsibilities at the top of the form below. Then conduct a WHS risk assessment and record your findings in the space provided.

Responsibilities of students under the Model WHS Act: s28

- Take reasonable care for your own health and safety by working safely at all times
- Take reasonable care to ensure that your acts or omissions don't put the health and safety of others at risk
- Follow all TAFE NSW WHS guidance and comply with all reasonable instructions from TAFE NSW staff to assist them in complying with the TAFE NSW WHS requirements
- In addition to the above, you must:
  - use and maintain machinery, tools and all other equipment properly and safely
  - ensure that your work area is free of hazards
  - notify a TAFE NSW staff member of actual or potential hazards
  - wear/use prescribed safety equipment
  - take notice of any safety signs and adhere to their instructions

Risks involved in this activity include:	**Control measures:**
Trip hazards (eg students bags) Objects dropped on feet (while equipment is taken to and from workbenches). Others: ____________________ ____________________ ____________________	Move bags and other objects from walkways Plan lifting of equipment Other: ____________________ ____________________ ____________________

My signature here indicates that I have read and understand my responsibilities under the Model WHS Act s28 (detailed above). I have also conducted a risk assessment before undertaking this activity and have identified measures to control these risks and have implemented them.

Signature: ____________________ Date: ____________________

2. Gather the equipment needed for this exercise.

3. Wire the filter in Figure 1.

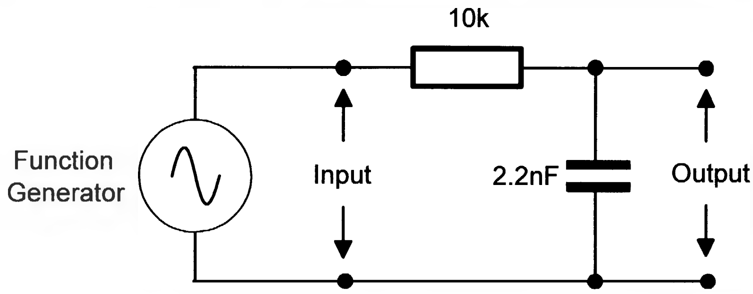


Figure 1

4. Connect the CRO's channel 1 input to the filter's input.

5. Connect the CRO's channel 2 input to the filter's output.

6. Adjust the function generator to output a 1kHz sinewave at <u>exactly</u> 10Vp-p.

7. Measure the output voltage and record this in the appropriate space on Table 1.

8. Calculate and record the circuit's gain in decibels using the equation: $Av_{(dB)} = 20Log\left(\frac{Vout}{Vin}\right)$.

9. Measure and record the phase difference (in degrees) between the two signals (if any). If you're not sure how to do this, read the instructions below.

Measuring phase difference
To measure the phase difference between two signals using the CRO:

1. Make sure both traces are centred.

2. Count the number of divisions for one period of the input signal.

2. Count the number of divisions between the two signals at the zero line.

3. Use the following equation to calculate the phase difference:

$$\Phi\ shift = \frac{\text{Number of divisions between the two signals}}{\text{Number of divisions for the period of the input}} \times 360^o$$

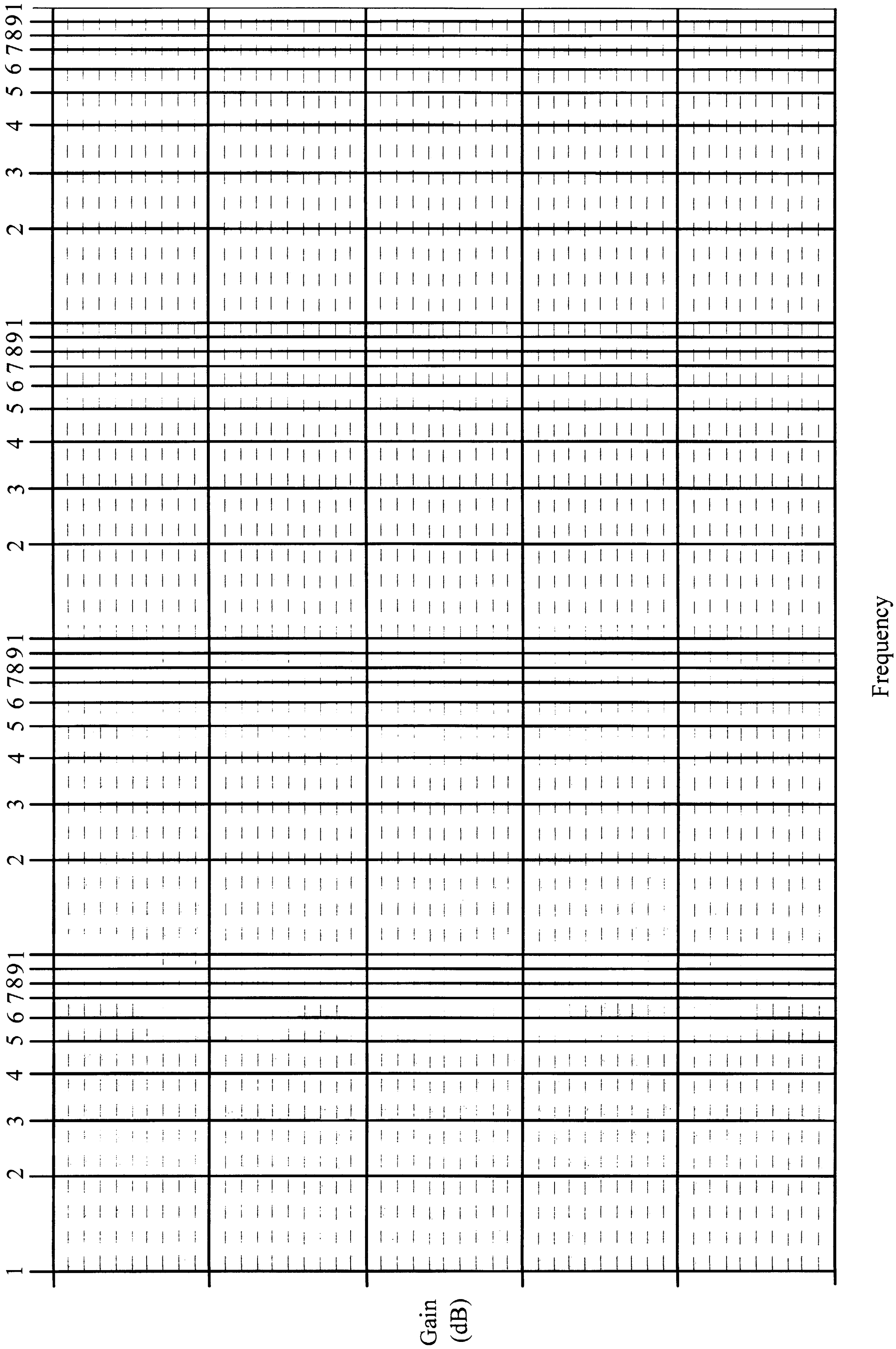
10. Determine whether the output signal is leading or lagging the input signal and note this in Table 1.

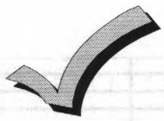
11. Repeat steps 7 to 10 for the remaining frequencies listed in Table 1. Be as accurate as possible with your measurements and don't measure the phase shift for frequencies where this cell has been greyed-out.

Table 1

Frequency (Hz)	Output voltage (peak-to-peak)	Gain (in decibels)	Phase shift	The output is leading/lagging
500				
1k				
3k				
5k				
6k				
6,500				
7k				
7,500				
8k				
9k				
10k				
13k				
15k				
20k				
30k				
60k				
100k				
200k				

12. Graph the frequency versus output in decibels on the log/lin graph paper below.





The teacher needs to check your work at this point...

13. Use your graph to find the approximate cut-off frequency (*fc*) and record this in Table 2.

14. Find the cut-off frequency precisely by measuring the frequency where the output voltage drops to 0.707 of the pass-band's output voltage. Record this value in Table 2 also.

15. Calculate and record the theoretical cut-off frequency (*fc*) of your filter.

Table 2

The cut-off frequency reading off the graph	The measured cut-off frequency	The theoretical cut-off frequency

Question 1
How close were your graphed, measured and calculated values of cut-off frequency? Explain the differences (other than component tolerances).

Question 2
Determine the approximate phase difference between the input and output signals at the cut-off frequency using your results in Table 1.

Question 3
Compare the value of phase shift found for question 2 with the theoretical value of phase shift at *fc*. Are they the same?

Question 4
What's the difference between the gain in decibels at 30kHz and 60kHz?

Question 5
What's the significance of this difference?

Question 6
Recalculate the roll-off in decibels/octave using 10kHz and 20kHz.

Question 7
Compare the value of roll-off you have calculated in questions 4 and 6. Are they the same? What accounts for their difference? (**Note:** The answer's not component tolerances!)

Question 8
What precaution must you take when determining the slope of the roll-off of a filter?

Question 9
Determine the slope of the roll-off in decibels/decade using appropriate frequencies in Table 1 (Note: remember the precaution you provided for question 8)

Question 10
Compare the measured value of roll-off in question 9 with the theoretical value. Are they the same?

Question 11
Why are 6kHz and 60kHz inappropriate frequencies to use to determine the roll-off of this filter in decibels/decade?

The teacher needs to check your work at this point...

16. Wire the filter in Figure 2.

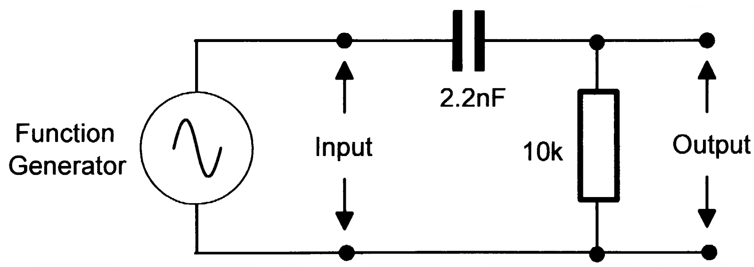


Figure 2

17. Connect the CRO's channel 1 input to the filter's input.

18. Connect the CRO's channel 2 input to the filter's output.

19. Adjust the function generator to output a 100kHz sinewave at exactly 10Vp-p.

20. Measure the output voltage and record this in the appropriate space on Table 3 (on the next page).

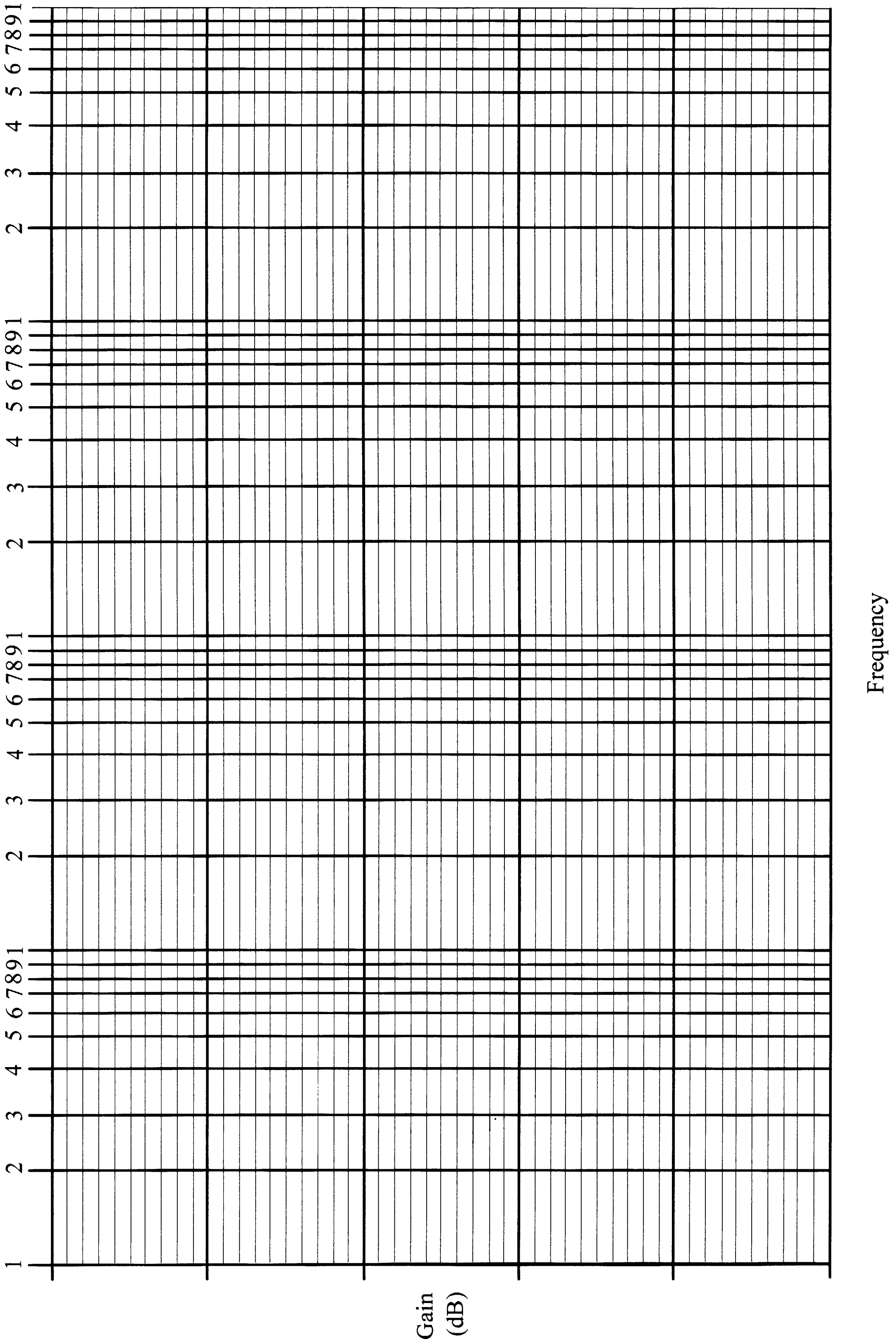
21. Calculate and record the circuit's gain in decibels using the equation: $Av_{(dB)} = 20Log\left(\frac{Vout}{Vin}\right)$.

22. Repeat steps 20 and 21 for the remaining frequencies listed in Table 3 being as accurate as possible with your measurements.

Table 3

Frequency (Hz)	Output voltage (peak-to-peak)	Gain (in decibels)
1k		
3k		
4k		
5k		
6k		
6,500		
7k		
7,500		
8k		
8,500		
9k		
10k		
13k		
15k		
20k		
30k		
60k		
100k		
120k		

23. Graph the frequency versus output in decibels on the log/lin graph paper below.



The teacher needs to check your work at this point...

24. Use your graph to find the approximate cut-off frequency (*fc*) and record this in Table 4.

25. Find the cut-off frequency precisely by measuring the frequency where the output voltage drops to 0.707 of the pass-band's output voltage. Record this value in Table 4 also.

26. Calculate and record the cut-off frequency (*fc*) of your filter.

Table 4

The cut-off frequency reading off the graph	The measured cut-off frequency	The theoretical cut-off frequency

Question 12
Compare these cut-off frequency values with those in Table 2. Why is the cut-off frequency of this circuit the same as the first circuit?

____

____

The teacher needs to check your work at this point...

Review questions

Answer these questions to check your understanding of what you have learnt for this chapter. Doing this will also help to prepare you for the tests.

1. Draw the circuit of a passive 1st order RL low-pass filter.

2. Calculate the cut-off frequency (*fc*) if the resistor in your filter is 2k2Ω and the inductor is 4H.

____

____

3. What is the phase difference between the output and the input signals at the cut-off frequency? Indicate whether the output would be leading or lagging the input.

____

____

4. Sketch the graph of frequency versus output voltage for your filter given the input voltage is 5Vp-p. Show on the graph the following: maximum output voltage; the cut-off frequency; the voltage at the cut-off frequency; and the slope of the roll-off.

5. Calculate the value of inductor that would give the filter a cut-off frequency (*fc*) of 12kHz.

6. Draw the circuit of a 1st order RL high-pass filter.

7. Calculate the cut-off frequency (*fc*) if the resistor in your filter is 8k2Ω and the inductor is 250mH.

8. What is the phase difference between the output and the input signals at the cut-off frequency? Indicate whether the output would be leading or lagging the input.

9. Calculate the value of resistor that would give the filter a cut-off frequency (*fc*) of 40kHz.

10. Draw the circuit of a 1st order RC high-pass filter.

11. Calculate the cut-off frequency if the resistor in your filter is 4k7Ω and the capacitor is 2.2nF.

12. What is the phase difference between the output and the input signals at the cut-off frequency? Indicate whether the output would be leading or lagging the input.

13. Sketch the graph of frequency versus output voltage for the circuit given the input voltage is 9Vp-p. Show on the graph the following: maximum output voltage; the cut-off frequency; the voltage at the cut-off frequency; and the slope of the roll-off.

14. What would happen to the cut-off frequency of your filter if the capacitor drifted high in value due to a fault?

15. Draw the circuit of a 1st order RC low-pass filter.

16. Calculate the cut-off frequency if the resistor in your filter is 10kΩ and the capacitor is 330pF.

17. What is the phase difference between the output and the input signals at the cut-off frequency? Indicate whether the output would be leading or lagging the input.

18. Calculate the value of capacitor that would give the filter a cut-off frequency of 100kHz.

19. What would happen to the cut-off frequency of your filter if the resistor drifted high in value due to a fault?

Section 5 Resonance

Purpose

To develop your understanding of how capacitors and inductors behave in electrical circuits when they are connected together.

Objectives

At the end of this section you should be able to:

- Define resonance as it relates to electrical circuits
- Draw a series and parallel RLC circuit
- Describe the frequency characteristics of series and parallel RLC circuits
- Draw the graphs of current and total circuit impedance versus frequency for series and parallel RLC circuits
- Describe the conditions of series and parallel RLC circuits at the resonant frequency
- Calculate the resonant frequency of series and parallel RLC circuits
- Calculate the voltage across the components in an RLC circuit for a given frequency
- Explain the terms *voltage magnification effect*, *Q-factor* and *damped oscillation*
- Calculate the *Q-factor* of an RLC circuit given all component values
- Observe the voltage across a suitable component in a resonant circuit using an oscilloscope while sweeping through a range of frequencies and use your observations to identify the resonant frequency experimentally
- Measure the voltage across a suitable component in a resonant circuit using an oscilloscope at a number of given frequencies to use the measurements to develop a frequency response graph

Introduction

Although it is possible to create band-pass and band-stop filters by connecting an RC or RL low-pass and high-pass filter together, the performance of such circuits is far from ideal and the range of possible centre frequencies is limited. To achieve better band-pass and band-stop frequency performance characteristics we must make use of the capacitor and inductor together.

When capacitors and inductors are connected either in series or parallel they exhibit an effect known as *resonance*. Resonance lends itself perfectly to band-pass and band-stop filters. In this section we'll examine the principles of resonance in detail so that we can apply these to band-pass and band-stop filters in Section 6.

What is resonance?

Have you ever been singing in the shower and noticed that singing the right note produces a sound that is louder and more powerful than the others even though you're not singing any louder? Or, have you ever run your finger around the top of glass at just the right speed and caused the glass to ring loudly? These are examples of resonance. Put simply, resonance is the effect whereby a stimulus of a given amount causes a much bigger reaction at one frequency than any other frequency.

Opera singers can make dramatic use of resonance by singing a particular note to cause a glass to break. Singing any note causes a glass to vibrate a little. However, if the note is at the glass's resonant frequency it vibrates much much more. If the note is loud enough the glass can shatter. All physical objects have a resonant frequency whether they be glasses, bells or bridges.

An electrical equivalent of resonance occurs in circuits wherever a capacitor and an inductor are connected together (provided the DC resistance in the circuit is not too large). The resonant frequency of LC circuits is the frequency where a maximum amount of an electrical property (such as current) occurs for a given input voltage (this point will be demonstrated graphically shortly). Capacitors and inductors can be connected in series to produce *series resonance* or in parallel to produce *parallel resonance*.

Series resonance

Figure 1 below shows an example of a series RLC circuit. The circuit does not need the resistor for the resonance effect to occur but one is usually shown anyway because inductors are never ideal and always have some DC resistance that affects the circuit's performance.

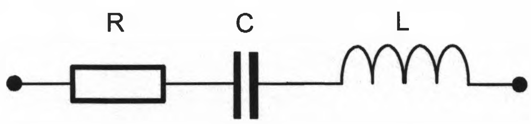


Figure 1 *Series RLC circuit*

To understand how the circuit operates and what happens at resonance you must draw on your understanding of the operation principles of capacitors and inductors. Recall that the reactance of a capacitor and inductor vary with frequency as shown in Figure 2. At relatively low frequencies the reactance of a capacitor is relatively large compared to the reactance of an inductor. At relatively high frequencies, the reactance of a capacitor is relatively small compared to the reactance of an inductor.

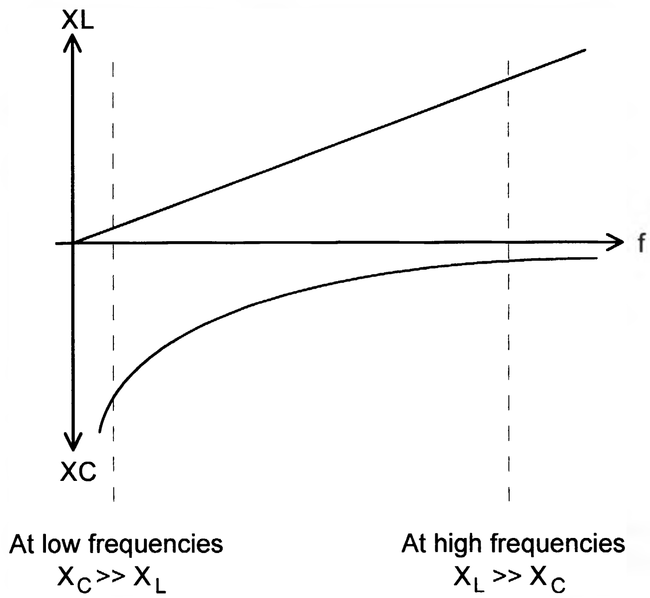


Figure 2 *Graph of frequency versus capacitive and inductive reactance*

As you may recall, because current leads voltage in capacitors and current lags voltage in inductors, the two reactances operate in opposition to each other (which is why X_L and X_C point in opposite directions on the same axis). That being the case, when capacitors and inductors are connected together, the smaller of the two reactances subtracts from the other.

At relatively low frequencies the total reactance is capacitive because the capacitive reactance is so much larger than the inductive reactance. Conversely, at relatively high frequencies, the total reactance is inductive because the capacitive reactance is so much smaller than the inductive reactance.

Importantly, at one crucial frequency, the reactances are equal (see Figure 3) and, because the reactances are in opposite directions, they cancel each other out and the total reactance is zero.

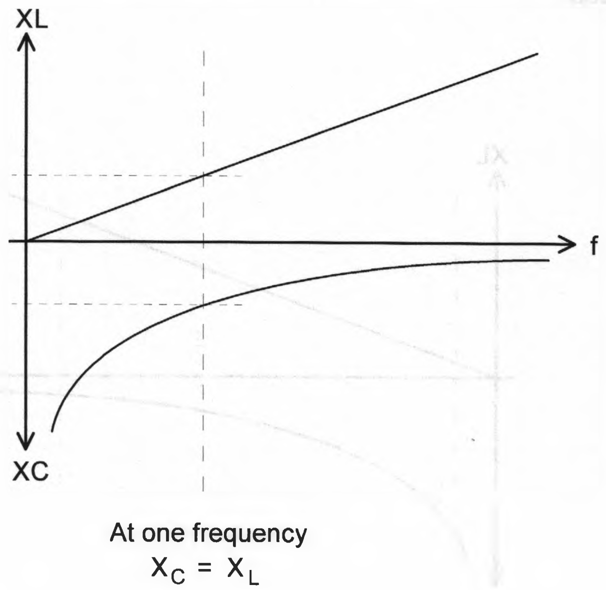


Figure 3 *At one frequency X_C and X_L are equal in magnitude and, because they are opposite in directions, they cancel each other out totally*

The significance of this for the series RLC circuit in Figure 1 becomes apparent when we consider the total circuit impedance and the circuit current.

Recall that impedance is the combination of all the resistances and reactances in a circuit and is calculated using the equation $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At relatively low frequencies the total circuit impedance is relatively large and capacitive because X_C is bigger than X_L. As frequency increases, inductive reactance gets bigger but capacitive reactance drops by a greater amount and so the overall circuit impedance goes down.

At relatively high frequencies the total circuit impedance is also relatively large but inductive because X_L is bigger than X_C. As frequency decreases, capacitive reactance gets bigger but inductive reactance drops by a greater amount and so the circuit impedance goes down.

Importantly, at the frequency where X_C and X_L are equal and opposite, they cancel each other out and the total circuit impedance is smallest and purely resistive. If we plot the graph of frequency versus total circuit impedance for the circuit in Figure 1 we get the graph in Figure 4 on the next page.

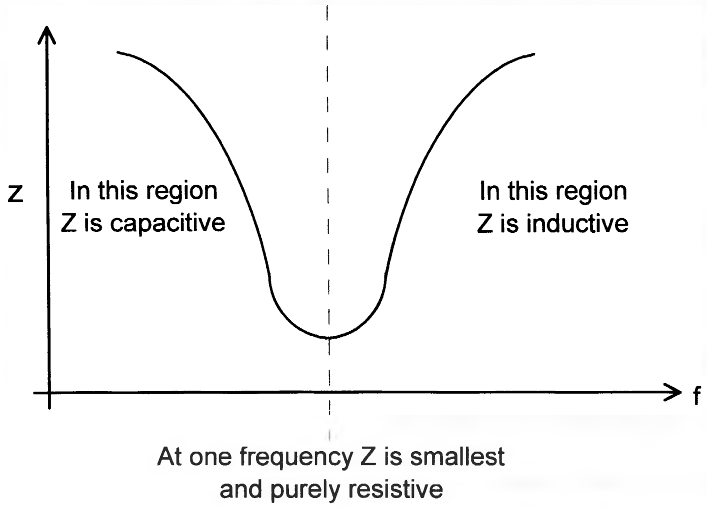


Figure 4 *Graph of frequency versus total circuit impedance in a series RLC circuit*

As we know, the current in a circuit is inversely proportional to impedance. When the total circuit impedance is maximum, the current is minimum. Conversely, when the total circuit impedance is minimum, the current is maximum. If we plot the graph of frequency versus current for the series RLC circuit of Figure 1 we the graph in Figure 5 below:

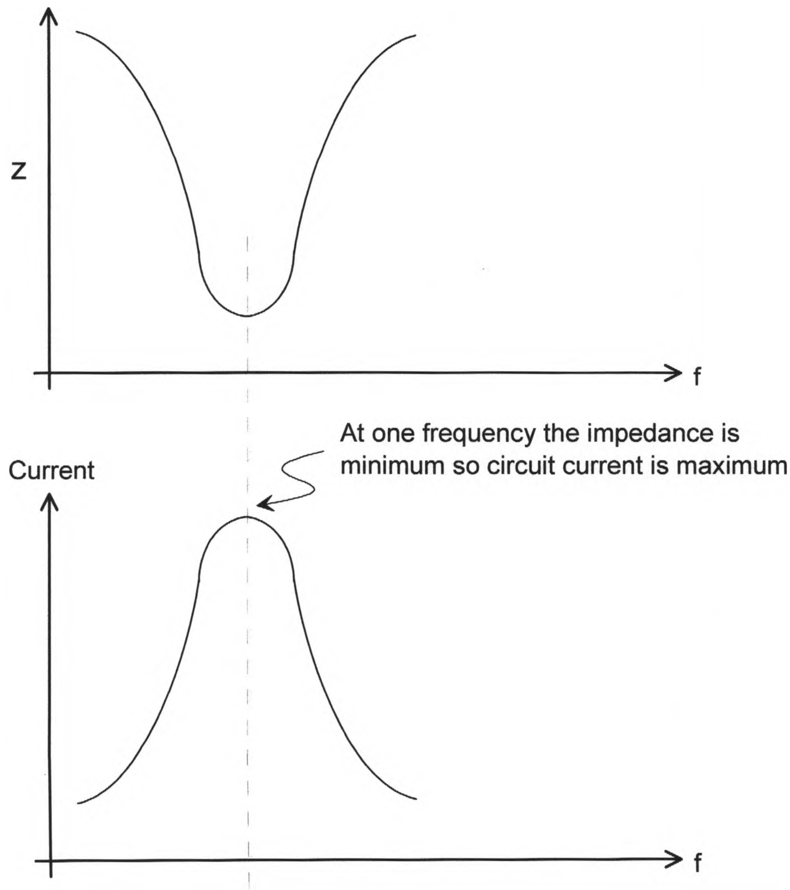


Figure 5 *The relationship between circuit impedance and circuit current*

Recall that resonance has been defined here as the effect whereby a stimulus of a given amount causes a much bigger reaction at one frequency than any other frequency. We can see a resonance effect occurring in the operation of an LC circuit. At one particular frequency the circuit current is much more than at any of the other frequencies. That's why this frequency is called the *resonant frequency*.

We can summarise the conditions of series resonance as follows:

- X_C and X_L are equal and opposite values cancelling each other out
- The total circuit impedance is purely resistive
- The total circuit impedance is minimum
- The circuit current is maximum

Calculating the resonant frequency

As we have seen, the capacitive and inductive reactances of an RLC circuit are equal and opposite at resonance. This gives us an opportunity to develop an equation for finding the theoretical resonant frequency. Recall that the theoretical values for X_L and X_C are calculated using the equations: $X_L = 2\pi fL$ and $X_C = \frac{1}{2\pi fC}$.

At resonance, we can say that:

$$2\pi fL = \frac{1}{2\pi fC}$$

Multiplying both sides by $2\pi fC$ gives:

$$(2\pi)^2 f^2\, LC = 1$$

Dividing both sides by $(2\pi)^2 LC$ gives:

$$f^2 = \frac{1}{(2\pi)^2 CL}$$

Squaring rooting both sides gives:

$$f = \sqrt{\frac{1}{(2\pi)^2 LC}}$$

This is the same as:

$$f = \frac{\sqrt{1}}{\sqrt{(2\pi)^2 LC}}$$

Which can be simplified to:

$$\boxed{fr = \frac{1}{2\pi\sqrt{LC}}}$$

Practise using the equation by trying the following questions.

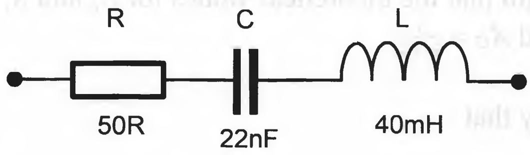


Figure 6

1. What is the resonant frequency of the circuit in Figure 6?

2. What would be the new resonant frequency if the inductor's value was changed to 82µH?

3. What is the value of the inductor in Figure 7 **below** if the circuit's resonant frequency is 4MHz?

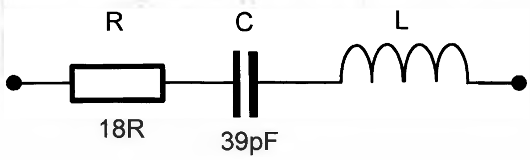


Figure 7

Voltage magnification

A curious effect occurs in series RLC circuits whereby at some frequencies the voltage across the capacitor and/or inductor is bigger than the applied voltage. This effect is known as *voltage magnification* and can be demonstrated with an example.

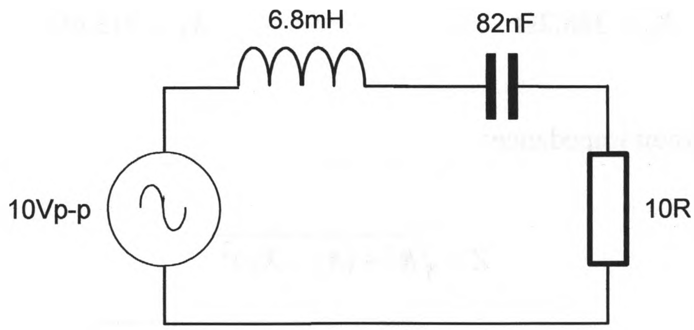


Figure 8

The circuit of Figure 8 shows a series RLC circuit connected to a 10Vp-p AC supply. The potential difference across the three components isn't fixed (like it would be if the circuit was just three resistors) because the reactances of the capacitor and the inductor are frequency dependent. To find the potential differences you need to nominate a frequency then follow the procedure below:

- Find X_C and X_L at the nominated frequency using $X_C = \frac{1}{2\pi f C}$ and $X_L = 2\pi f L$
- Find the total circuit impedance using $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- Find the circuit current using $I = \frac{E}{Z}$
- Find the voltage across the individual components using $V_C = I \times X_C$, $V_L = I \times X_L$ and $V_R = I \times R$

Let's do an example at 5kHz. First, find the capacitive and inductive reactances:

$$X_C = \frac{1}{2\pi f C} \qquad X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi \times 5kHz \times 82nF} \qquad X_L = 2\pi \times 5kHz \times 6.8mH$$

$$X_C = 388.2\Omega \qquad X_L = 213.6\Omega$$

Next, find the total circuit impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{10^2 + (213.6\Omega - 388.2\Omega)^2}$$

$$Z = \sqrt{30585.16}$$

$$Z = 174.9\Omega$$

Then find the circuit current:

$$I = \frac{E}{Z}$$

$$I = \frac{10Vp\text{-}p}{174.9\Omega}$$

$$I = 57.18mAp\text{-}p$$

Finally, find the potential difference across the three components:

$$V_C = I \times X_C \qquad V_L = I \times X_L \qquad V_R = I \times R$$

$$V_C = 57.18mA \times 388.2\Omega \qquad V_L = 57.18mA \times 213.6\Omega \qquad V_R = 57.18mA \times 10\Omega$$

$$V_C = 22.2Vp\text{-}p \qquad V_L = 12.2Vp\text{-}p \qquad V_R = 0.572Vp\text{-}p$$

From this example we can see that at 5kHz the voltage magnification effect occurs across both the capacitor and the inductor - their potential differences are both bigger than the supply voltage.

Table 1 shows the potential differences across the components calculated at thirteen other frequencies:

Table 1

Frequency (in Hz)	X_C (Ω)	X_L (Ω)	Z (Ω)	I (mA)	V_C (volts)	V_L (volts)	V_R (volts)
100	19,409.14	4.27	19,404	0.52	10	0	0.01
500	3,881.83	21.36	3,860	2.59	10.05	0.06	0.03
1,000	1,940.91	42.73	1,898	5.27	10.23	0.23	0.05
2,000	970.46	85.45	885.1	11.3	10.97	0.97	0.11
3,500	554.55	149.54	405.1	24.67	13.68	3.69	0.25
5,000	388.18	213.63	174.8	57.2	22.2	12.22	0.57
6,740	287.97	287.97	10	1,000	287.97	287.97	10
7,500	258.79	320.44	62.46	160	41.41	51.27	1.6
9,000	215.66	384.53	169.2	59.1	12.75	22.73	0.59
12,000	161.74	512.71	351.1	28.48	4.61	14.6	0.28
15,000	129.39	640.88	511.6	19.55	2.53	12.53	0.2
25,000	77.64	1,068.14	990.6	10.09	0.78	10.78	0.1
50,000	38.82	2,136.28	2,098	4.77	0.19	10.19	0.05
100,000	19.41	4,272.57	4,253	2.35	0.05	10.04	0.02

Notice that the voltage magnification effect occurs at many frequencies but is greatest at the resonant frequency (6740Hz in the case of Figure 8).

As an aside, notice also that the other calculations at 6740Hz support the earlier discussion of the series RLC circuit's operation. The reactances at the resonant frequency are equal, the total circuit impedance is minimum (and the same value as the resistor because the reactances cancel each other out) and the circuit current is maximum (at 1A).

Q-factor

The voltage magnification effect provides us with means of determining the *quality* of RLC circuits. The greater the potential difference across the capacitor or inductor at resonance compared to the voltage across the DC resistive component in the circuit, the better the quality of the circuit (generally speaking!).

The quality of an RLC circuit can be expressed numerically and is known as *Q-factor* (or just *Q*). It can be found by using one of two equations.

$$\boxed{\text{Q-factor} = \frac{V_C}{V_R} \quad \text{or} \quad \text{Q-factor} = \frac{V_L}{V_R}}$$

Let's do an example with the values in Table 1. At resonance, the potential difference across the capacitor and resistor is 287.97V and 10V respectively. So, the Q-factor of the circuit in Figure 8 is:

$$\textit{Q-factor} = \frac{V_C}{V_R}$$

$$\textit{Q-factor} = \frac{287.97V}{10V}$$

$$\textit{Q-factor} = 28.8$$

The same Q-factor would have been produced if we had used the potential across the inductor instead of across the capacitor.

The main factor that affects the quality of an RLC circuit is DC resistance. The *Q* of the circuit can be improved by making the resistor smaller. To demonstrate this, let's find the new Q-factor for the circuit in Figure 8 (redrawn as Figure 9 on the next page) with the resistor changed to 5Ω.

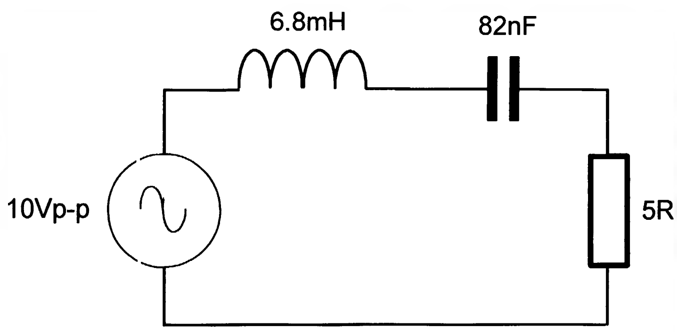


Figure 9

1. Calculate the circuit's impedance at resonance.

2. Use the impedance to calculate the circuit current at resonance.

3. Calculate the reactance of either the inductor or capacitor at resonance.

4. Calculate the potential difference across the capacitor/inductor and the resistor at resonance.

5. Calculate the circuit's Q-factor.

You should have found that halving the value of the resistor in the circuit doubled its Q-factor to 57.6.

Obviously, the smaller the resistor's value, the bigger the circuit's Q. Furthermore, removing the resistor altogether gives the best Q. However, removing the resistor does not remove all of the resistance from the circuit because inductors always have some DC resistance as well as reactance. In fact, that's the reason the resistor is shown in the circuit in the first place: often it is not a real component. If a high Q-factor is needed for an RLC circuit then an inductor with a very low DC resistance must be used.

The example on the previous page demonstrates that finding the Q-factor for the circuit requires several steps. The process can be made shorter with the help of a little algebra.

We know that:

$$Q = \frac{V_C}{V_R} \qquad \text{and} \qquad Q = \frac{V_L}{V_R}$$

And since $V = I \times R$ we can say that:

$$Q = \frac{I \times X_C}{I \times R} \qquad \text{and} \qquad Q = \frac{I \times X_L}{I \times R}$$

These expressions can be simplified to:

$$Q = \frac{X_C}{R} \qquad \text{and} \qquad Q = \frac{X_L}{R}$$

Substituting for X_C and X_L in the expressions give:

$$Q = \frac{\frac{1}{2\pi fc}}{R} \qquad \text{and} \qquad Q = \frac{2\pi fL}{R}$$

$$Q = \frac{1}{2\pi fCR}$$

If these two expressions are multiplied together we get:

$$Q^2 = \frac{2\pi f L}{2\pi f C R^2}$$

And this can be simplified as follows:

$$Q^2 = \frac{L}{CR^2}$$

$$\sqrt{Q^2} = \sqrt{\frac{L}{CR^2}}$$

$$Q = \sqrt{\frac{L}{C}} \times \sqrt{\frac{1}{R^2}}$$

Which ultimately simplifies to:

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Let's put the equation to the test for the circuit in Figure 8:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{10}\sqrt{\frac{6.8mH}{82nF}}$$

$$Q = \frac{1}{10}\sqrt{82927}$$

$$Q = 28.79$$

Parallel resonance

Figure 10 shows a parallel RLC circuit. As with the series resonant circuit, it doesn't need the resistor to work but one is shown anyway because the inductor has a DC resistive component that affects the circuit's operation.

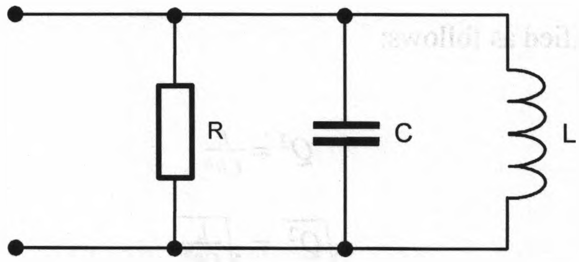


Figure 10 *Parallel RLC circuit*

To understand how the circuit operates and what happens at resonance you must again draw on your understanding of the operation of capacitors and inductors. As the graph in Figure 11 shows, at relatively low frequencies the reactance of a capacitor is relatively large compared to the reactance of an inductor. At relatively high frequencies, the reactance of a capacitor is relatively small compared to the reactance of an inductor. At one frequency - the resonant frequency - the two reactances are equal but opposite.

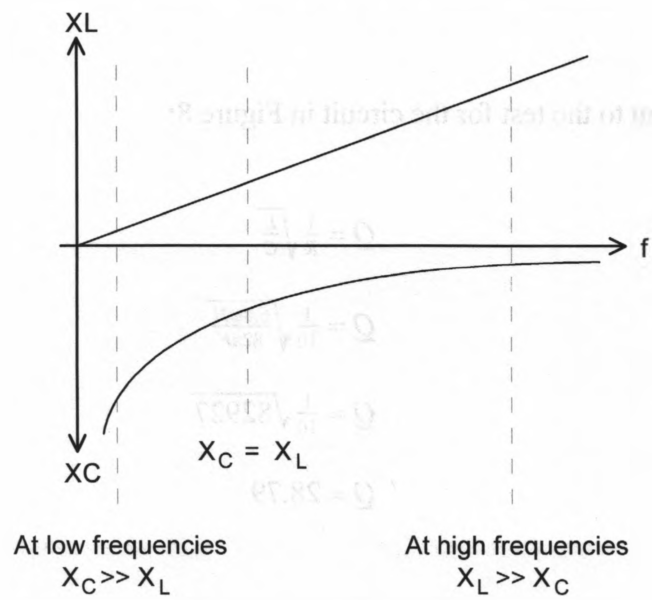


Figure 11 *Graph of frequency versus capacitive and inductive reactance*

The operation of capacitors and inductors in parallel RLC circuits is exactly the same as in series RLC circuits. However, the effect that they have on the circuit's impedance (Z) is different because the components are in parallel.

As you know, when resistors are connected in parallel the total resistance is lower than the smallest resistance in the parallel network. The same is true of reactive components connected in parallel: the total circuit impedance is lower than the smallest reactance. That being the case, at relatively low frequencies the inductor has a relatively small reactance so the circuit impedance is relatively small. Under these conditions the circuit is said to be inductive.

At relatively high frequencies, the capacitor has a relatively small reactance so the circuit impedance is relatively small. Under these conditions the circuit is said to be capacitive. At the resonant frequency, the reactance of the inductor and capacitor cancel each other out so the circuit impedance is the same as the resistance and is maximum. This is shown in Figure 12 below.

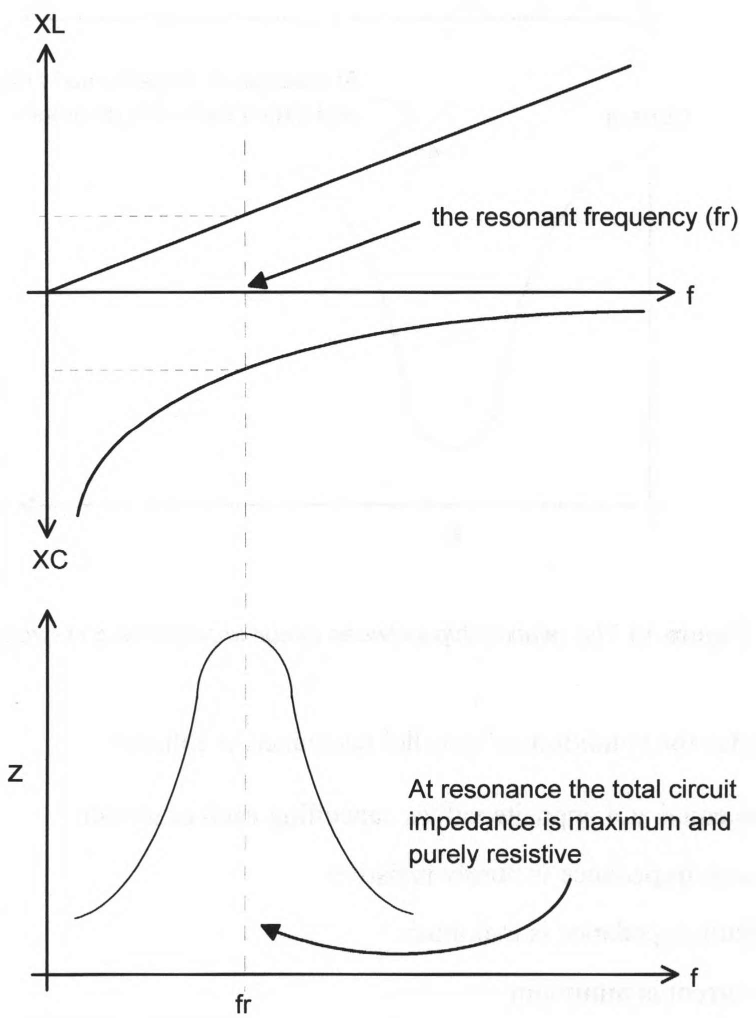


Figure 12 *Graph of frequency versus circuit impedance*

As you know, the current in a circuit is inversely proportional to impedance. When the total circuit impedance is maximum, the current is minimum. Conversely, when the total circuit impedance is minimum, the current is maximum. If we plot the graph of frequency versus current for the parallel RLC circuit of Figure 10 we get:

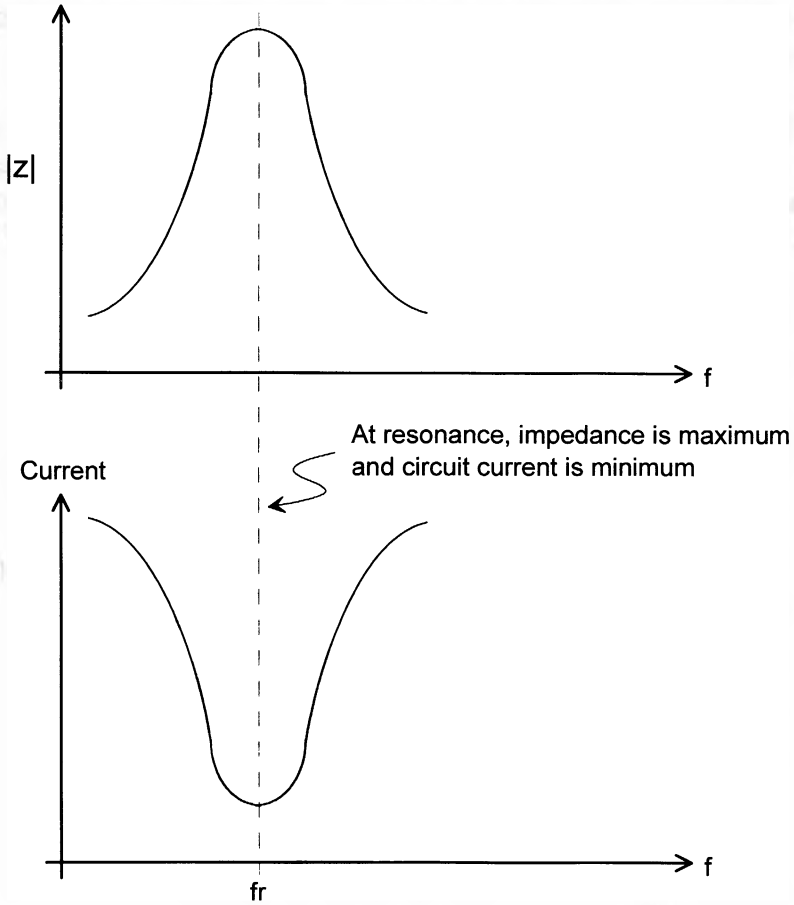


Figure 13 *The relationship between circuit impedance and circuit current*

We can summarise the conditions of parallel resonance as follows:

- X_C and X_L are equal and opposite values cancelling each other out
- The total circuit impedance is purely resistive
- The total circuit impedance is maximum
- The circuit current is minimum

The equation for finding the resonant frequency is the same as for series RLC circuits. The equation for finding the Q-factor is a little different and is: $Q = R\sqrt{\frac{C}{L}}$.

Damped oscillations

The parallel RLC circuit is often known as a *tank circuit*. To understand why, consider a parallel LC circuit made with an ideal inductor and capacitor (see Figure 14). Being ideal, the components don't have any resistance (which is why a resistor isn't shown).

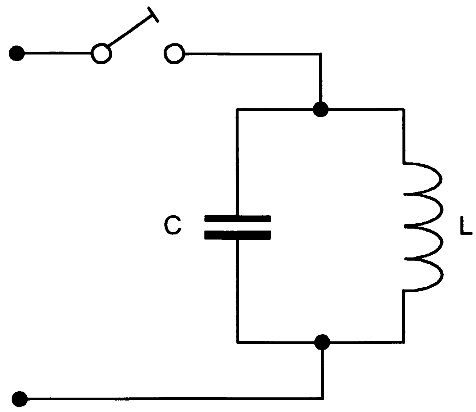


Figure 14 *A parallel LC circuit made from an ideal inductor and capacitor*

When the switch is closed, current flows through the capacitor and charges it up instantly (see Figure 15).

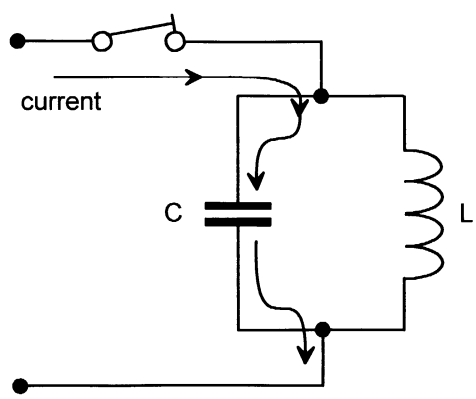


Figure 15 *When the switch is closed current flows through the capacitor and charges it up instantly*

When the switch is opened the inductor looks like a short-circuit across the capacitor so it discharges through it (see Figure 16). This builds up an electromagnetic field about the inductor in the process.

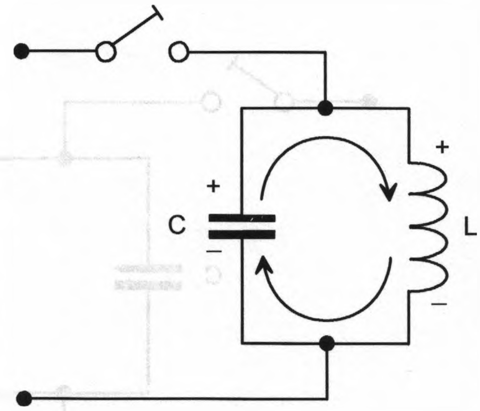


Figure 16 *The capacitor discharges through the inductor*

When the capacitor is fully discharged, the circuit current stops so the electromagnetic field about the inductor collapses producing a back EMF and current flows in the reverse direction charging the capacitor back up (see Figure 17).

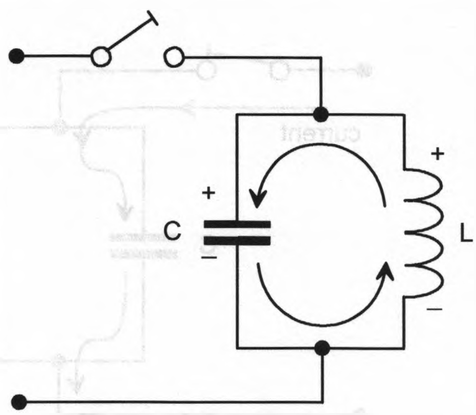


Figure 17 *The collapsing field about the inductor charges the capacitor back up*

When the electromagnetic field has fully collapsed, the capacitor is fully charged but again the inductor looks like a short-circuit connected across it. So, the capacitor discharges through the inductor building up an electromagnetic field that eventually collapses and charges the capacitor back up. Because the two components in the circuit are ideal, there are no losses and this process continues indefinitely. In this way, the circuit stores energy the way a water tank stores water.

The charge and discharge characteristics of the inductor and capacitor means that the voltage developed across the parallel network while this is going on is a sinewave. Losses in either the inductor or capacitor result in the capacitor charging up a little less each cycle until eventually, there is no energy left in the circuit. The greater the losses in the components, the quicker this occurs (see Figure 18).

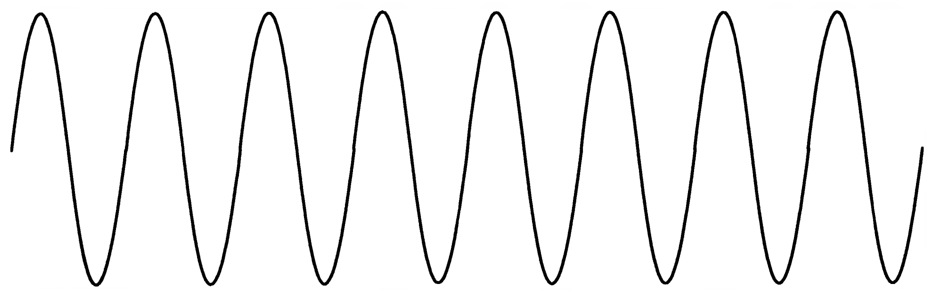


Figure 18a *The sinewave produced by the charging and discharging of the capacitor*

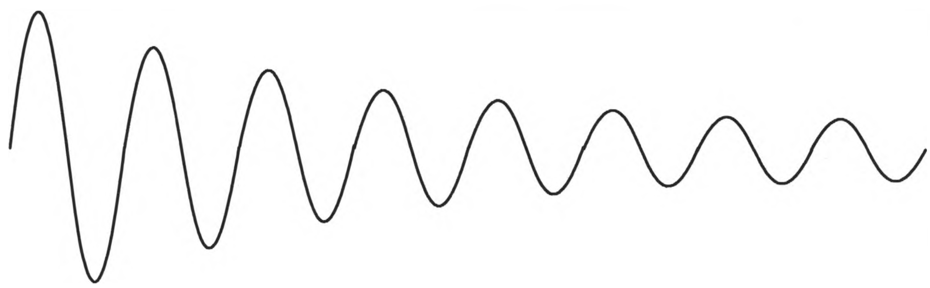


Figure 18b *Losses in the components causes the charging and discharging process to decay*

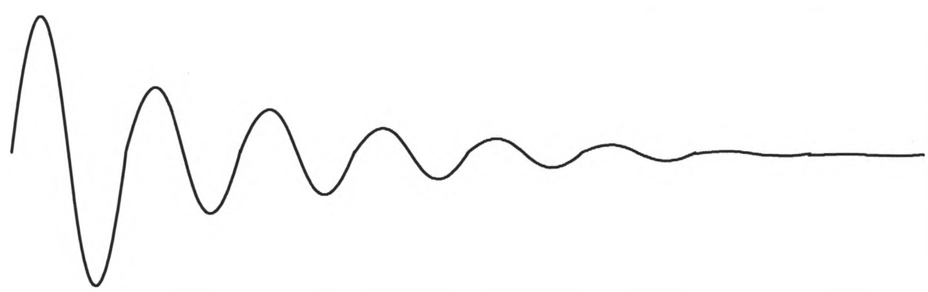


Figure 18b *The greater the losses the quicker the decay*

These decaying oscillations are known as *damped oscillations*. If the losses in the circuit (caused by the DC resistance) are great enough there is no oscillation at all.

Student notes

Skill practice 5

Practise measuring the voltage across components in resonant circuits using an oscilloscope

This exercise is practise for the sorts of skills you may be required to perform in a practical test. Remember, in any practical tests you'll be working alone so make sure that you can perform all the steps. It should take you about 1¼ hours to complete this exercise.

Equipment

- Interface panel
- 1kΩ ¼W resistor
- 2.5mH inductor
- 820pF capacitor
- three BNC to banana-plug leads
- banana leads

Remember:

Follow TAFE NSW WHS guidance at all times!

Work tasks

1. Read your WHS responsibilities at the top of the form below. Then conduct a WHS risk assessment and record your findings in the space provided.

Responsibilities of students under the Model WHS Act: s28

- Take reasonable care for your own health and safety by working safely at all times
- Take reasonable care to ensure that your acts or omissions don't put the health and safety of others at risk
- Follow all TAFE NSW WHS guidance and comply with all reasonable instructions from TAFE NSW staff to assist them in complying with the TAFE NSW WHS requirements
- In addition to the above, you must:
  - use and maintain machinery, tools and all other equipment properly and safely
  - ensure that your work area is free of hazards
  - notify a TAFE NSW staff member of actual or potential hazards
  - wear/use prescribed safety equipment
  - take notice of any safety signs and adhere to their instructions

Risks involved in this activity include:	**Control measures:**
Trip hazards (eg students bags) Objects dropped on feet (while equipment is taken to and from workbenches). Others: ______________________ ______________________ ______________________	Move bags and other objects from walkways Plan lifting of equipment Other: ______________________ ______________________ ______________________

My signature here indicates that I have read and understand my responsibilities under the Model WHS Act s28 (detailed above). I have also conducted a risk assessment before undertaking this activity and have identified measures to control these risks and have implemented them.

Signature: ______________________ Date: ______________________

2. Gather the equipment needed for this exercise.

3. Wire the circuit of Figure 1. Make sure that the components are wired in the order shown.

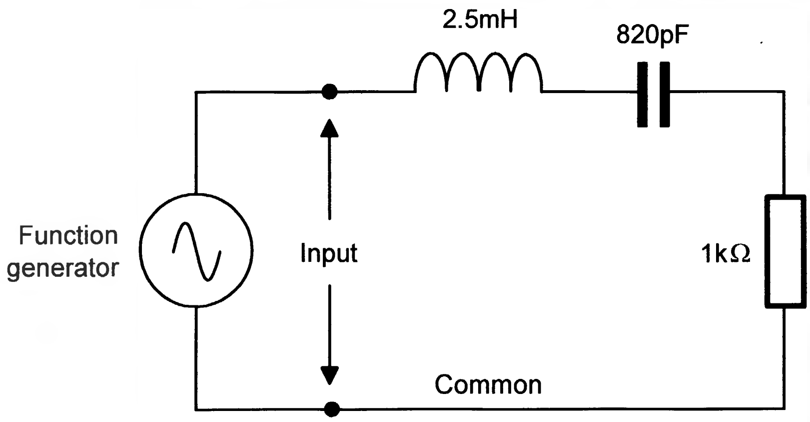


Figure 1

4. Connect the CRO's channel 1 input to the circuit's input. Make sure the CRO lead's black alligator clip is connected to the circuit's common.

5. Adjust the function generator to output a sinewave at about 75kHz.

6. Set the function generator's output amplitude to exactly 20Vp-p.

7. Connect the CRO's channel 2 input across the resistor. Make sure the CRO lead's black alligator clip is connected to the circuit's common.

8. Vary the input signal's frequency up and down a little and note the changes on the resistor's potential difference.

9. Adjust the input signal's frequency until the resistor's potential difference is maximum. Record the signal's frequency in the Table 1 in the first column's blank cell.

10. Accurately measure then record the resistor's potential difference.

11. Use the resistor's potential difference to calculate the circuit's current. **Tip:** Use Ohm's Law.

12. Use the circuit current and the function generator's output voltage to calculate the circuit's impedance. **Tip:** Use Ohm's Law to do this too.

13. Repeat steps 10 to 12 for all the given frequencies listed in Table 1.

    Tip: When you measure the resistor's voltage, make sure that you adjust the CRO's channel 2 *Vertical Attenuation* control to make the wave as big on the screen as possible. Remember, this improves the accuracy of your measurements.

Table 1

Frequency (Hz)	Potential difference	Circuit current	Circuit impedance
10k			
20k			
30k			
40k			
50k			
60k			
70k			
80k			
90k			
120k			
150k			
200k			
300k			
400k			
500k			
600k			
700k			
800k			

The teacher needs to check your work at this point...

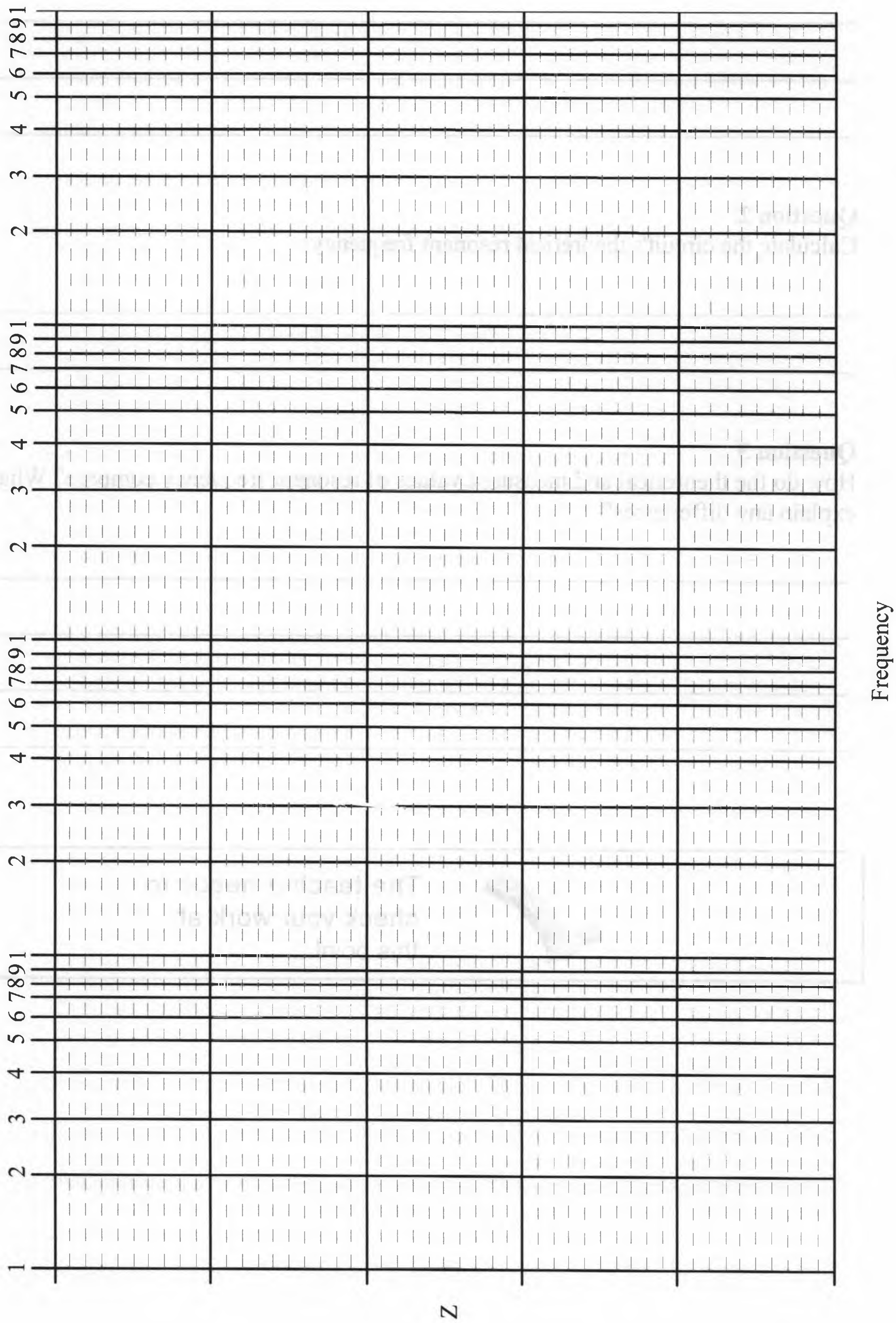
Question 1
What's the significance of the frequency where the circuit current is highest?

Question 2
Calculate the circuit's theoretical resonant frequency.

Question 3
How do the theoretical and measured values of resonant frequency compare? What might explain any differences?

The teacher needs to check your work at this point...

14. Plot the graph of frequency versus impedance on log/lin graph paper below.



Question 4

In what way would this graph to be different if the components where connected in parallel instead of in series?

The teacher needs to check your work at this point...

Student notes

Review questions

Answer these questions to check your understanding of what you have learnt for this chapter. Doing this will also help to prepare you for the tests.

1. The impedance of a series RLC circuit at the resonant frequency is

   - ☐ capacitive.
   - ☐ purely resistive and maximum.
   - ☐ inductive.
   - ☐ purely resistive and minimum.

2. The impedance of a series RLC circuit at frequencies below the resonant frequency is

   - ☐ capacitive.
   - ☐ purely resistive and maximum.
   - ☐ inductive.
   - ☐ purely resistive and minimum.

3. The impedance of a series RLC circuit at frequencies above the resonant frequency is

   - ☐ capacitive.
   - ☐ purely resistive and maximum.
   - ☐ inductive.
   - ☐ purely resistive and minimum.

4. The circuit current of a series RLC circuit at the resonant frequency is

   - ☐ maximum.
   - ☐ infinite.
   - ☐ minimum.
   - ☐ zero.

Questions 5 to 12 refer to the circuit of Figure 1

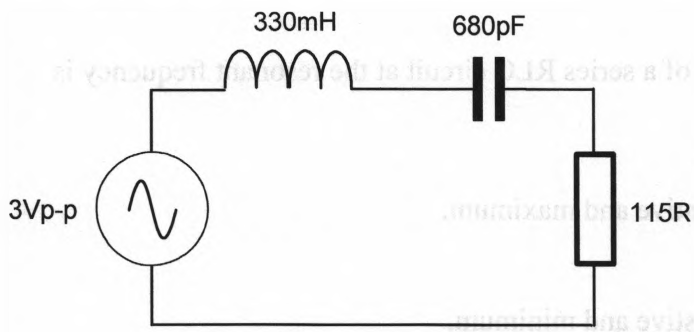


Figure 1

5. Calculate the resonant frequency.

6. What is the circuit impedance at resonance?

7. Calculate the circuit current at resonance.

8. Calculate the voltage across the inductor at resonance. **Tip:** Calculate the inductor's reactance first.

9. What is the voltage across the resistor at resonance?

10. Use the answers to questions 8 and 9 to find the circuit's Q-factor.

11. Calculate the Q-factor. **Tip:** There are two Q-factor equations so make sure you use the correct one for a series resonant circuit.

12. Calculate the value of capacitor that would give the circuit in Figure 2 below a resonant frequency of 75kHz.

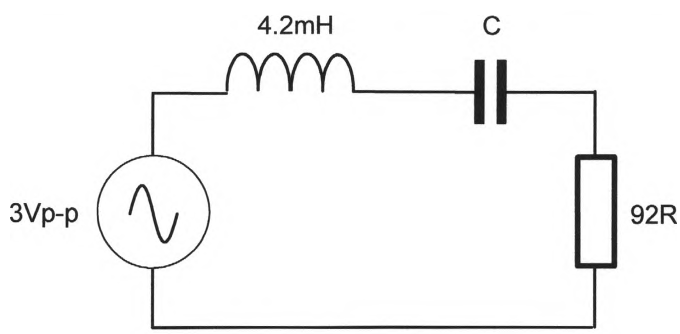


Figure 2

13. The impedance of a parallel RLC circuit at the resonant frequency is

- ☐ capacitive.
- ☐ purely resistive and maximum.
- ☐ inductive.
- ☐ purely resistive and minimum.

14. The impedance of a parallel RLC circuit at frequencies below the resonant frequency is

- ☐ capacitive.
- ☐ purely resistive and maximum.
- ☐ inductive.
- ☐ purely resistive and minimum.

15. The impedance of a parallel RLC circuit at frequencies above the resonant frequency is

- ☐ capacitive.
- ☐ purely resistive and maximum.
- ☐ inductive.
- ☐ purely resistive and minimum.

16. The circuit current of a parallel RLC circuit at the resonant frequency is

- ☐ maximum.
- ☐ infinite.
- ☐ minimum.
- ☐ zero.

Questions 17 to 19 refer to the circuit of Figure 3

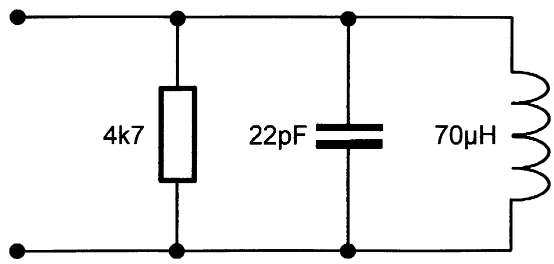


Figure 3

17. Calculate the resonant frequency.

18. What is the total circuit impedance at resonance?

19. Calculate the circuit's Q-factor. **Tip:** Use the Q-factor equation for parallel resonant circuits.

20. Calculate the value of inductor that would give the circuit in Figure 4 below a resonant frequency of 630kHz.

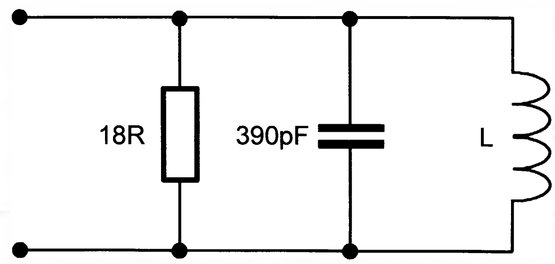


Figure 4

21. What is another name for a parallel LC circuit?

Section 6 Band-pass and band-stop filter circuits

Purpose

To develop your ability to identify passive and active filters in electronic systems and recognise whether or not they are operating correctly.

Objectives

At the end of this section you should be able to:

- Identify passive RCL band-pass and band-stop filter circuits
- Calculate the centre frequency of passive RCL band-pass and band-stop filter circuits
- State the phase relationship in degrees between input and output voltages at the centre frequency for passive RCL band-pass and band-stop filter circuits
- Define the term *selectivity*
- Calculate the centre frequency, bandwidth or Q-factor of a band-pass and band-stop filter given any two of these parameters
- Perform frequency response tests on simple RLC band-pass and band-stop filters using an oscilloscope

Introduction

Section 5 explained how series and parallel RLC circuits exhibit a resonance effect. When reading these notes you probably noticed that the graphs of impedance and current versus frequency looked like the frequency response of practical band-pass and band-stop filters. In fact, RLC circuits are ideally suited for these applications. In this section we'll look at how RLC circuits can be used to make practical band-pass and band-stop filters.

The passive series RLC band-pass filter

Figure 1 shows an example of an RLC band-pass filter. In this circuit, the resistor shown is an actual component across which the output voltage is developed. It should not be confused with the series resistance that all series combinations of *L* and *C* exhibit (referred to on page 5-2)

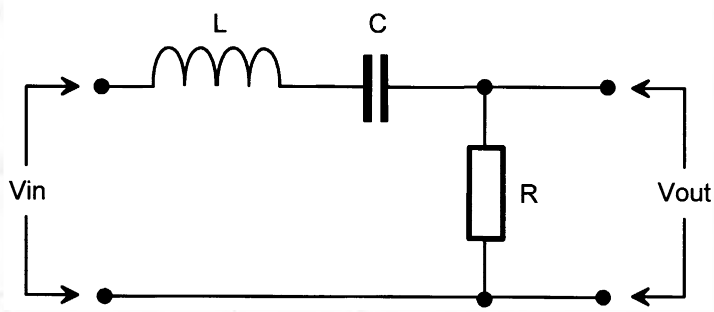


Figure 1 *RLC band-pass filter*

To explain the circuit's operation, recall that in a series RLC circuit, the current changes with frequency as shown in Figure 2.

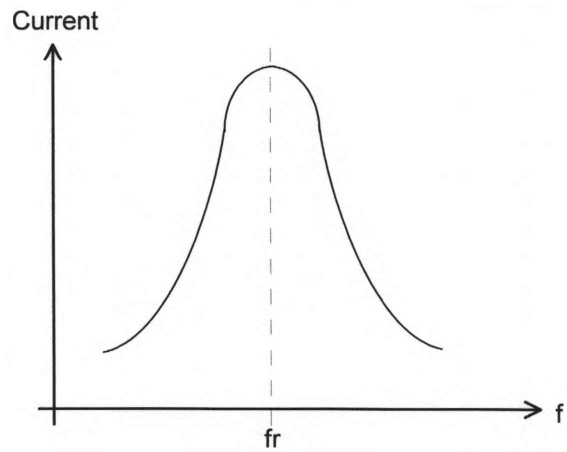


Figure 2 *Graph of frequency versus current in a series RLC circuit*

As the current through a resistor and the potential difference across it are directly proportional to each other (Ohm's law), then when current in the circuit of Figure 1 is small the potential difference across the resistor is also small. Similarly, when the current is large, the potential difference across the resistor is also large. This means that the potential difference across the resistor changes with frequency as shown in Figure 3.

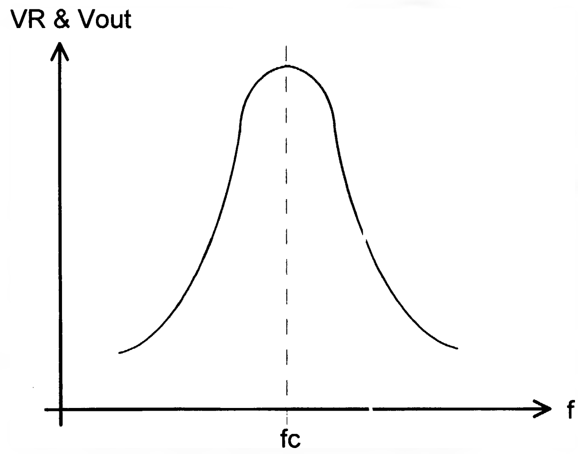


Figure 3 *The graph of frequency versus the potential difference across the resistor in Figure 1*

As the potential difference across the resistor is also the output voltage, it can be seen that the circuit is a band-pass filter.

The passive parallel RLC band-pass filter

A parallel RLC circuit can also be used as a band-pass filter as shown in Figure 4. In this circuit the resistor shown is an actual component and should not be confused with the parallel resistance that all parallel combinations of *L* and *C* exhibit (referred to on page 5-16).

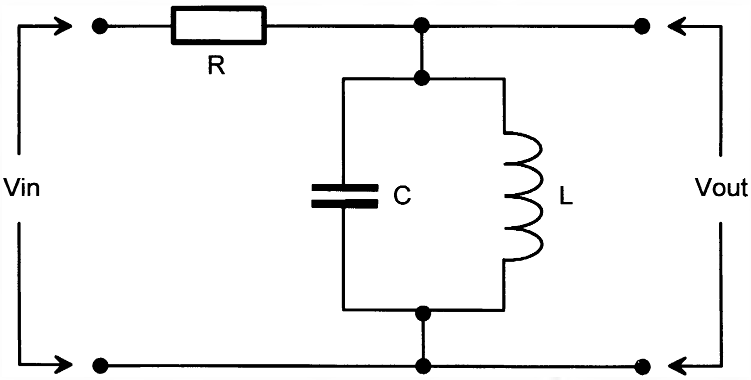


Figure 4 *Parallel RLC band-pass filter*

To explain the circuit's operation, recall that in a parallel RLC circuit, the impedance changes with frequency as shown in Figure 5.

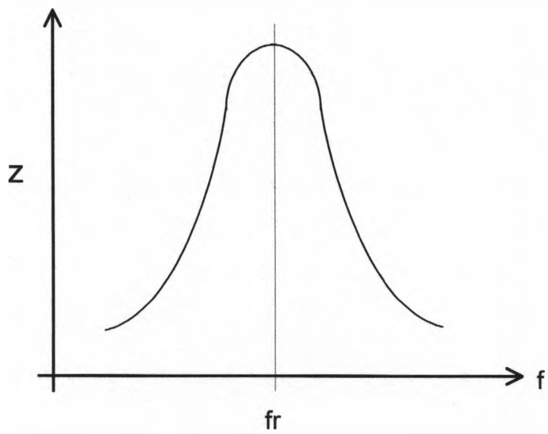


Figure 5 *The graph of frequency versus impedance in a parallel RLC circuit*

That being the case, the change in potential difference across the network with changes in frequency looks like that shown in Figure 6 below.

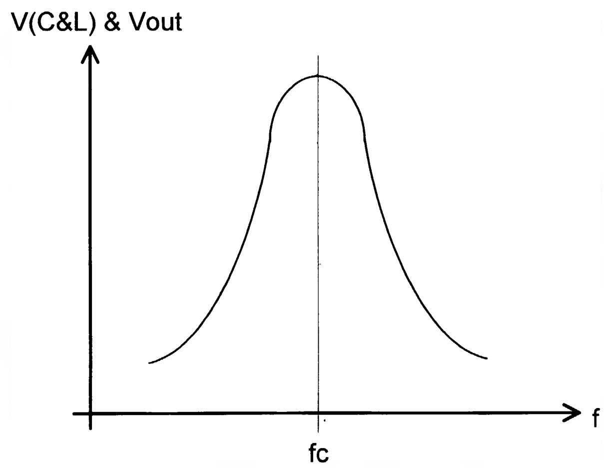


Figure 6 *The voltage dropped across the parallel capacitor-inductor network which is also the output voltage*

As the potential difference across the parallel LC network is also the output voltage, we can see that the circuit operates as a band-pass filter.

The passive series RLC band-stop filter

Figure 7 shows a series RLC circuit being used as a band-stop filter.

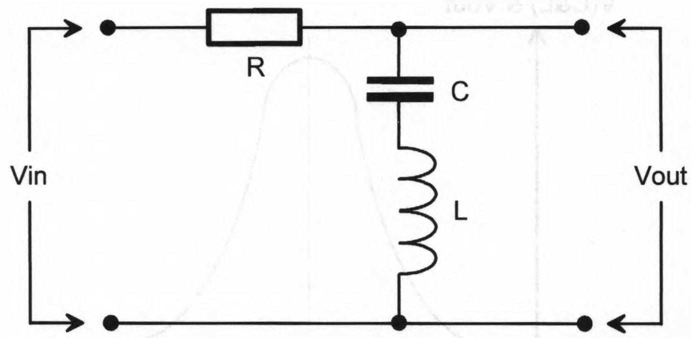


Figure 7 *RLC band-stop filter*

To explain the circuit's operation, recall that in a series RLC circuit, the impedance changes with frequency as shown in Figure 8.

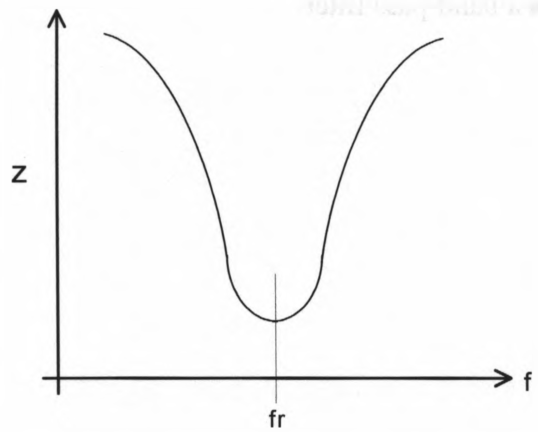


Figure 8 *The graph of frequency versus impedance for a series RLC circuit*

That being the case, changes in the potential difference across the series LC combination with changes in frequency looks like that in Figure 9 below.

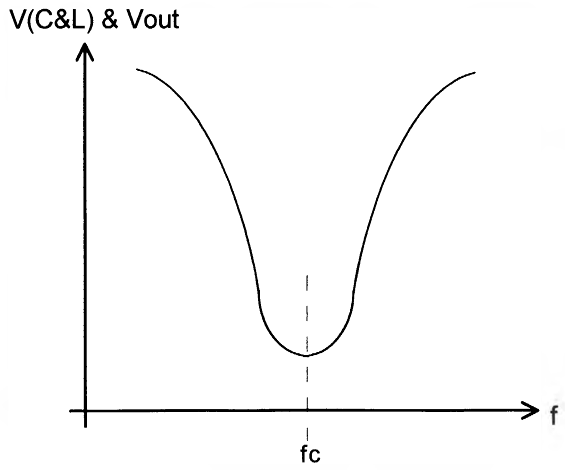


Figure 9 *The potential difference across the series capacitor-inductor network*

As the potential difference across the series LC network is also the output voltage, we can see that the circuit operates as a band-stop filter.

The passive parallel RLC band-stop filter

A parallel RLC circuit can also be used as a band-stop filter as shown in Figure 10.

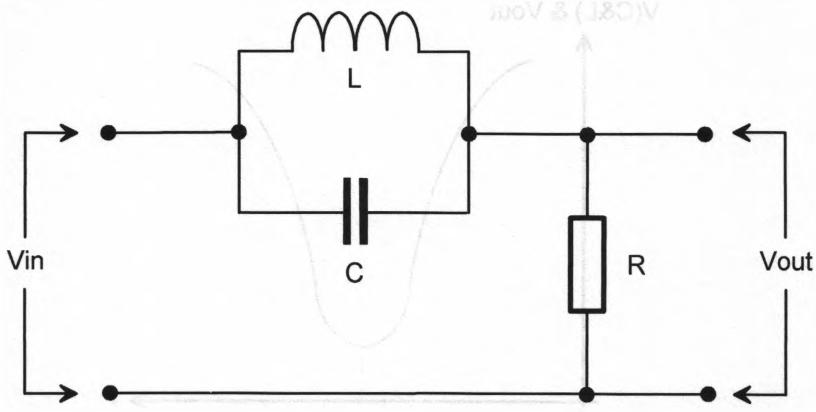


Figure 10 *Parallel RLC band-stop filter*

To explain the circuit's operation, recall that in a parallel RLC circuit, the current changes with frequency as shown in Figure 11.

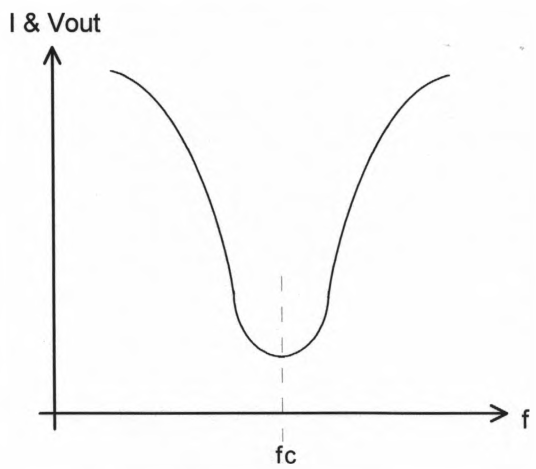


Figure 11 *Graph of frequency versus circuit current in a parallel capacitor-inductor network*

That being the case, the change in potential difference across the resistor with changes in frequency will look like that shown in Figure 12 below.

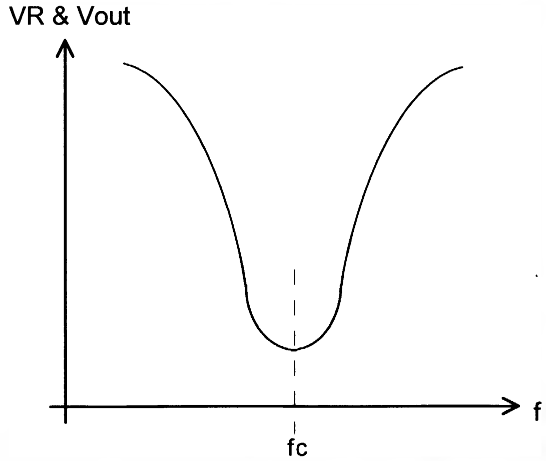


Figure 12 *The potential difference across the resistor*

As the potential difference across the resistor is also the output voltage, we can see that the circuit operates as a band-stop filter.

Calculating the centre frequency of RLC band-pass and band-stop filters

As the centre frequency of both the series and parallel RLC band-pass and band-stop filters occurs at the circuit's resonant frequency, the theoretical centre frequency is found by using the equation:

$$\boxed{fc = \frac{1}{2\pi\sqrt{LC}}}$$

Output voltage phase shift

The phase shift of the output voltage of both the series and parallel band-pass and band-stop filters swings anywhere from almost -90° through 0° and almost up to +90°. The exact amount of phase shift depends on the input signal's frequency and the components' values.

You're not required to specify whether the output voltage is leading or lagging the input voltage above and below the resonant frequency or calculate by how much. However, you are expected to know that the phase shift is exactly 0° at the centre frequency. This can be proved using a phasor diagram but the best way to appreciate this is to remember that at resonance, an RLC circuit is purely resistive (for both series and parallel L-C combinations). And, as you know, in purely resistive circuits, input and output voltages are in phase, so the phase shift at the centre frequency is 0°.

Filter selectivity

The *selectivity* of a band-pass and band-stop filter is a reference to its bandwidth and the steepness of the skirt (roll-off). Filters with a relatively narrow bandwidth and a very steep skirt are more selective than filters with a relatively wide bandwidth and a gradual skirt.

The selectivity of band-pass and band-stop filters is directly proportional to the circuit's Q-factor. That is, filters with a high Q are more selective than filters with a low Q (see Figure 13).

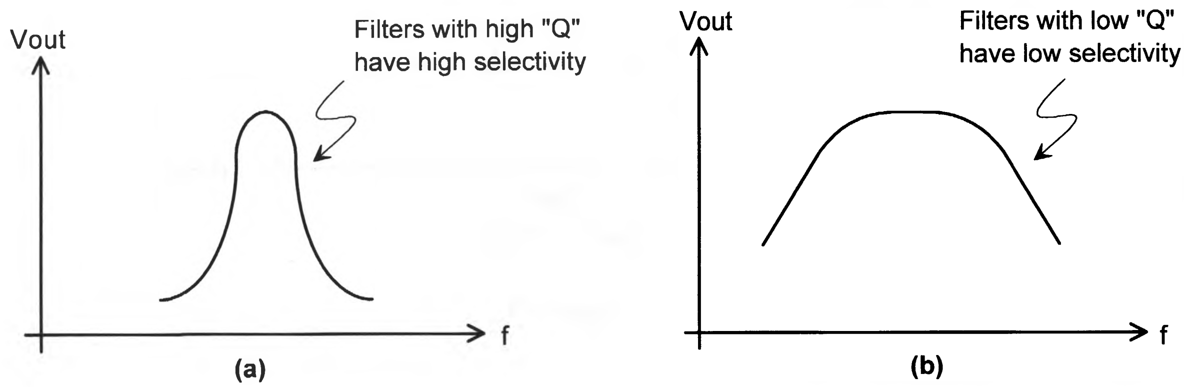


Figure 13 *Band-pass filter responses that are: a) very selective; and b) not very selective*

It is possible to calculate the bandwidth of a band-pass and band-stop filter if its centre frequency and the Q-factor are known:

$$\boxed{BW = \frac{fc}{Q}}$$

Where:
fc = centre frequency and
BW = bandwidth

Let's do an example. What is the bandwidth of a band-pass filter with centre frequency of 702kHz and a Q of 47?

$$BW = \frac{fc}{Q}$$

$$BW = \frac{702kHz}{47}$$

$$BW = 14.93kHz$$

By transposing the equation it's possible to find the Q-factor of a band-pass and band-stop filter if the centre frequency and bandwidth are known. For example, what is the Q-factor of the filter with the frequency response in Figure 14?

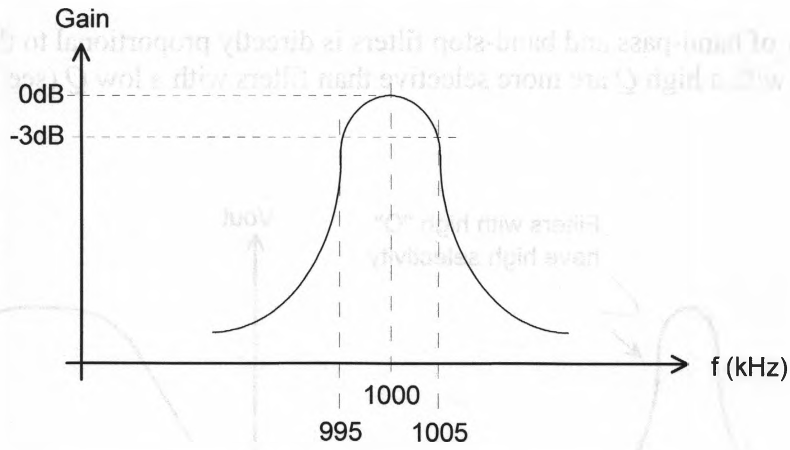


Figure 14

To solve this, we need two pieces of information: the centre frequency and the bandwidth. The centre frequency can be read straight off the graph and is 1,000kHz (or 1MHz). The bandwidth isn't given but $f1$ and $f2$ are. As bandwidth is found using the equation $BW = f2 - f1$, we can work out that the filter's bandwidth is 10kHz. Putting these values into the equation for finding Q-factor gives:

$$Q\text{-}factor = \frac{1000kHz}{10kHz}$$

$$Q\text{-}factor = 100$$

Typical figures for the Q-factor of RLC band-pass and band-stop filters range from 10 to about 100.

Practise finding the Q-factor of RLC circuits by trying the following questions for yourself.

1. Calculate the bandwidth of a band-pass filter with a centre frequency of 104.9MHz and a Q-factor of 700.

____

____

____

2. Calculate the Q-factor of a band-stop filter with a centre frequency of 150kHz, an *f1* of 144kHz and an *f2* of 156kHz. **Tip:** This is two calculations.

Increasing the selectivity

There are two approaches to increasing the selectivity of band-pass and band-stop filters. The first involves reducing the filter's bandwidth can by increasing their Q-factor. To that end, the DC resistance in the circuit must be reduced and/or the reactances increased.

The second approach to increasing the selectivity, which can be done instead of, or in addition to, reducing the bandwidth involves increasing the steepness of the filter's skirts and this can be done by adding reactive elements.

With that said, there is much, much more to the theory of filters implemented using inductors together with capacitors that goes well beyond trade level electronics. If you want to know more, Google "classical filter design", "Bessel filter", "Butterworth filter", "Chebyshev filter", and "elliptic filter".

Student notes

Skill practice 6

Practise measuring the output voltage of, and the phase shift introduced by, BPFs and BSFs using an oscilloscope

This exercise is practise for the sorts of skills you may be required to perform in a practical test. Remember, in any practical tests you'll be working alone so make sure that you can perform all the steps. It should take you about 1½ hours to complete this exercise.

Equipment

- Inductor panel
- 2k2Ω ¼W resistor
- 820pF capacitor
- three BNC to banana-plug leads
- banana leads

Remember:

Follow TAFE NSW WHS guidance at all times!

Work tasks

1. Read your WHS responsibilities at the top of the form below. Then conduct a WHS risk assessment and record your findings in the space provided.

Responsibilities of students under the Model WHS Act: s28

- Take reasonable care for your own health and safety by working safely at all times
- Take reasonable care to ensure that your acts or omissions don't put the health and safety of others at risk
- Follow all TAFE NSW WHS guidance and comply with all reasonable instructions from TAFE NSW staff to assist them in complying with the TAFE NSW WHS requirements
- In addition to the above, you must:
  - use and maintain machinery, tools and all other equipment properly and safely
  - ensure that your work area is free of hazards
  - notify a TAFE NSW staff member of actual or potential hazards
  - wear/use prescribed safety equipment
  - take notice of any safety signs and adhere to their instructions

Risks involved in this activity include:	**Control measures:**
Trip hazards (eg students bags) Objects dropped on feet (while equipment is taken to and from workbenches).	Move bags and other objects from walkways Plan lifting of equipment
Others: ____________________ ____________________ ____________________	Other: ____________________ ____________________ ____________________

My signature here indicates that I have read and understand my responsibilities under the Model WHS Act s28 (detailed above). I have also conducted a risk assessment before undertaking this activity and have identified measures to control these risks and have implemented them.

Signature: ____________________ Date: ____________________

2. Gather the equipment needed for this exercise.

3. Wire the circuit of Figure 1.

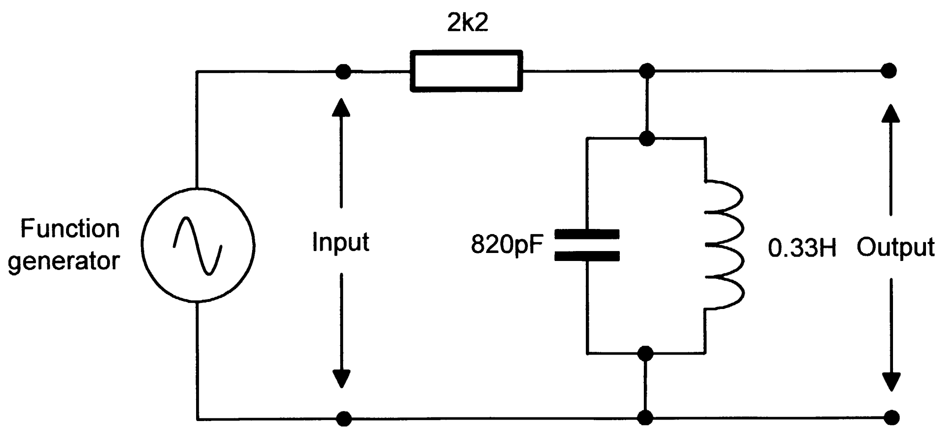


Figure 1

4. Connect the CRO's channel 1 input to the filter's input.

5. Connect the CRO's channel 2 input to the filter's output.

6. Adjust the function generator to output a 10kHz sinewave at exactly 10Vp-p.

7. Measure the filter's output voltage and record this in the appropriate space on Table 1 (on the next page).

8. Calculate and record the circuit's voltage gain in decibels using the equation: $Av_{(dB)} = 20Log\frac{Vout}{Vin}$.

9. Measure and record the phase difference between the two signals (for the frequencies where these cells are not shaded).

   Note: Indicate whether the output is leading or lagging by using the "+" and "-" symbols.

10. Repeat steps 7 to 9 for all the frequencies listed in Table 1 including the *f1* and *f2* frequencies.

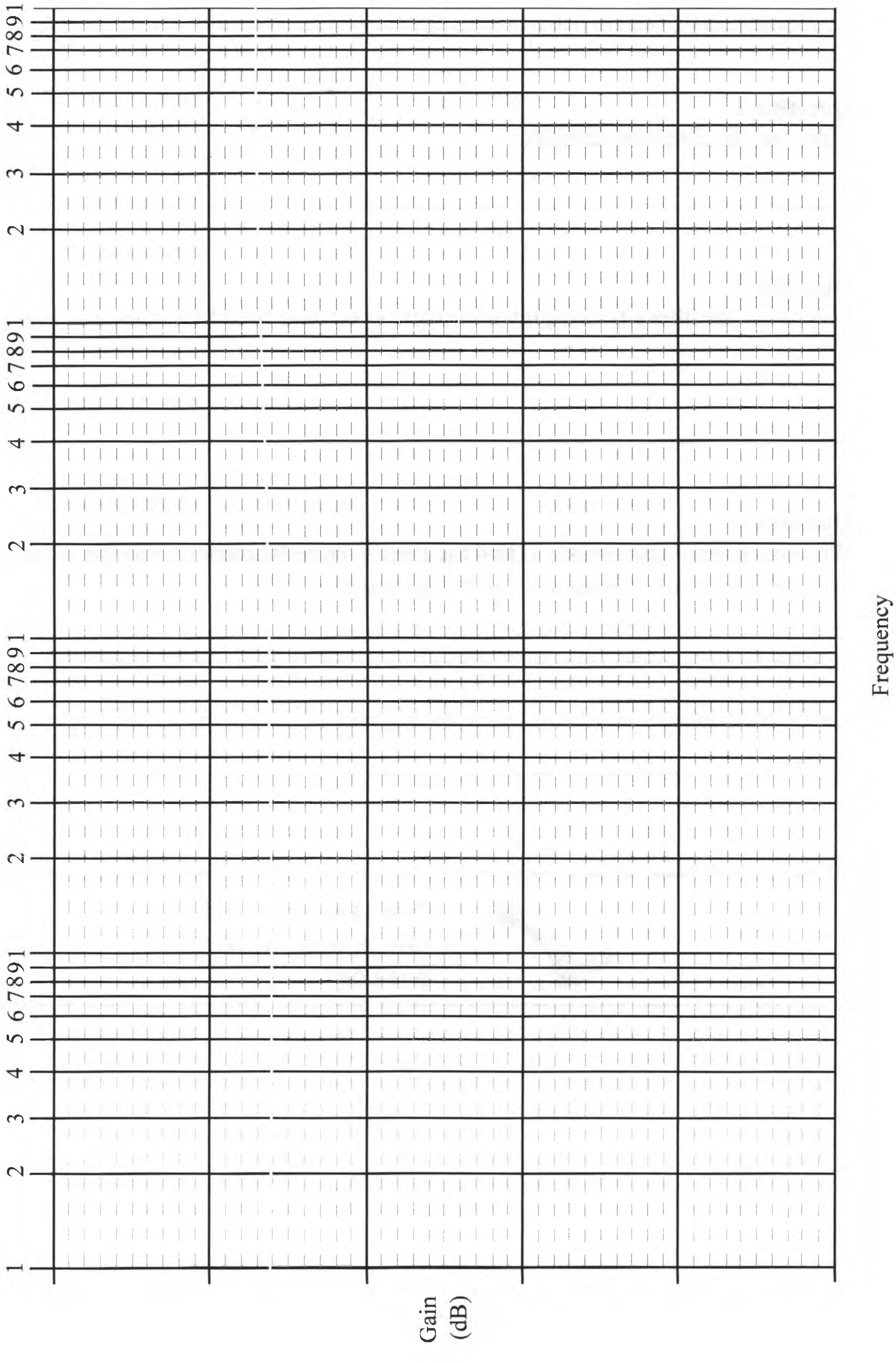
    Note 1: Be as accurate as possible with your measurements and don't measure the phase shift at frequencies where this cell has been greyed-out.

    Note 2: Remember, *f1* and *f2* occur at the frequency where the output voltage is 0.707 of the maximum output voltage.

Table 1

Frequency (Hz)	Output voltage (peak-to-peak)	Gain (in decibels)	Output Phase shift
100			
200			
500			
700			
f1 =			
2k			
5k			
7k			
10k			
20k			
50k			
f2 =			
100k			
200k			
500k			
700k			
1M			

11. Graph the frequency versus filter gain in decibels on the log/lin graph paper below.

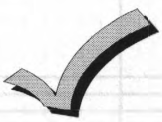


Question 1
Calculate the filter's centre frequency using: $fc = \sqrt{f1 \times f2}$.

Question 2
Calculate the filter's bandwidth.

Question 3
Determine the filter's Q using the information you have found for questions 1 and 2.

Question 4
Why was it more appropriate to find the filter's centre frequency using the equation given in Question 1 instead of using $fc = \frac{f2-f1}{2}$? **Tip:** See page 3-7.



The teacher needs to check your work at this point...

12. Wire the circuit of Figure 2.

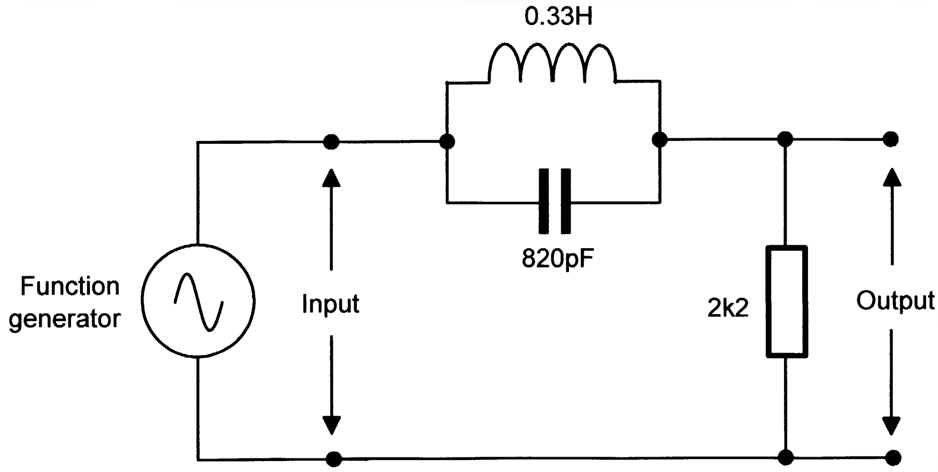


Figure 2

13. Connect the CRO's channel 1 input to the filter's input.

14. Connect the CRO's channel 2 input to the filter's output.

15. Adjust the function generator to output a 100Hz sinewave at exactly 10Vp-p.

16. Measure the filter's output voltage and record this in the appropriate space on Table 2 (on the next page).

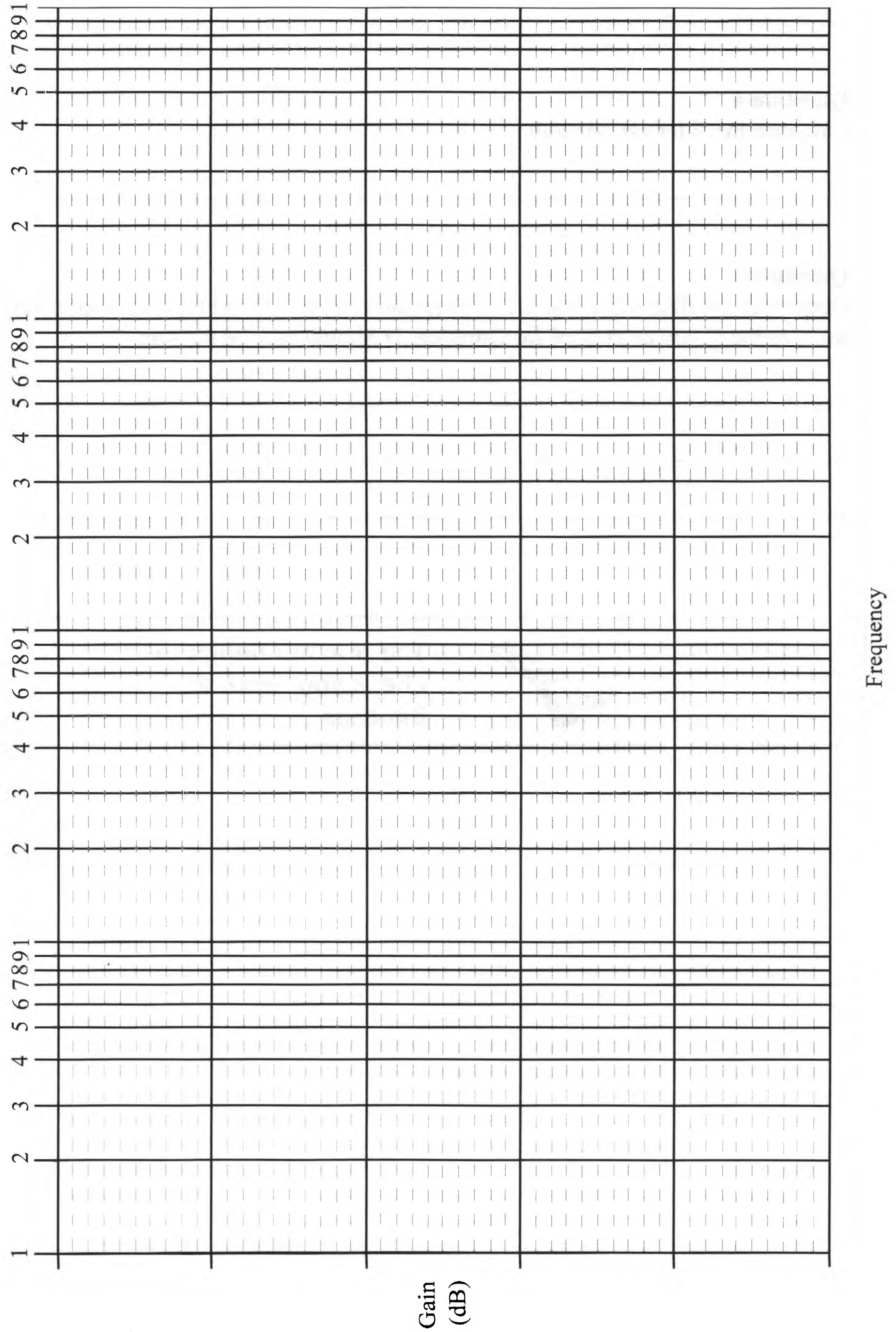
17. Calculate and record the circuit's gain in decibels.

18. Repeat steps 16 and 17 for all the frequencies listed in Table 2 being as accurate as possible with your measurements.

Table 2

Frequency (Hz)	Output voltage (peak-to-peak)	Gain (in decibels)
100		
200		
500		
700		
f1 =		
2k		
5k		
7k		
10k		
20k		
50k		
f2 =		
100k		
200k		
500k		
700k		
1M		

19. Graph the frequency versus output in decibels on the log/lin graph paper below.



Question 5
Calculate the filter's centre frequency using the equation given in Question 1.

Question 6
Calculate the filter's bandwidth.

Question 7
Determine the filter's Q using the information you have found for Questions 5 and 6. Is it the same as the Q you calculated for Question 3? Should they be the same?

The teacher needs to check your work at this point...

Review questions

Answer these questions to check your understanding of what you have learnt for this chapter. Doing this will also help to prepare you for the tests.

1. What is the relationship between the Q of a band-pass or band-stop filter and its selectivity?

2. What is the relationship between the Q of a band-pass or band-stop filter and its bandwidth?

3. What is the relationship between the Q of a band-pass or band-stop filter and the slope of its skirt?

4. State two ways of improving the selectivity of a band-pass or band-stop filter.

Questions 6 to 13 refer to Figure 1

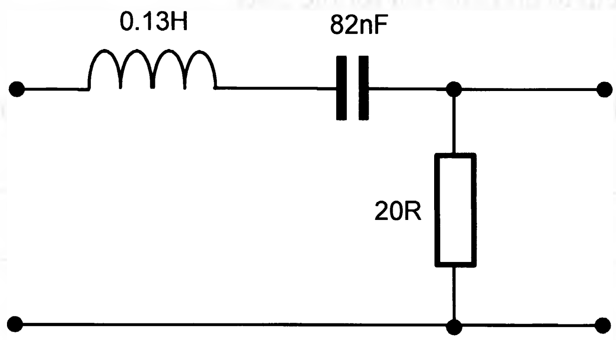


Figure 1

5. What type of filter is the circuit?

6. Why is the filter's output voltage much smaller than the input voltage at relatively low and high frequencies?

7. Why is the filter's output voltage maximum at the centre frequency?

8. Calculate the filter's centre frequency from component values.

9. Calculate the filter's Q-factor from component values.

10. Calculate the filter's bandwidth.

11. What is the phase relationship between the input and output signals at the centre frequency?

12. Calculate the circuit current at the resonant frequency if 10Vp-p is connected to the filter's input. [Tip: If you're not sure where to start with this question have another look at the conditions of series resonance on page 5-6.]

13. Calculate the output voltage at resonance if the circuit current is 150mA.

Questions 14 to 21 refer to the filter in Figure 2

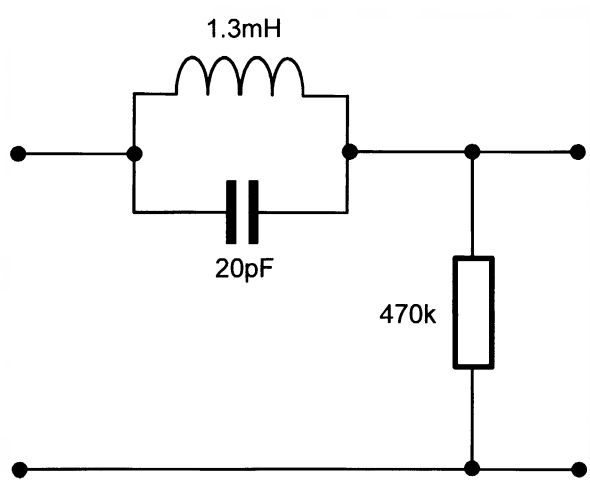


Figure 2

14. What type of filter is the circuit?

15. Why is the filter's output voltage maximum at relatively low and high frequencies?

16. Why is the filter's output voltage much lower at the centre frequency?

17. Calculate the filter's centre frequency from component values.

18. Calculate the filter's Q-factor from component values.

19. Calculate the filter's bandwidth.

20. What is the phase shift between the input and output voltages at the centre frequency?

21. Calculate value of Q necessary for the filter to have a bandwidth of 5kHz.

Appendix 1

Mathematical models of complex waveforms

Recall that the mathematical model of a pure sinewave is:

$$v = ASin\theta$$

This equation can be used to find the sinewave's *instantaneous voltage* (v). That is, the sinewave's voltage at any point in the cycle if its angle (θ) and the signal's peak voltage (A) are known.

If the angle at which the instantaneous voltage occurs isn't known it can be calculated provided the period of the waveform (P) and the time at which the instantaneous voltage occurs (t) are known. This information can be substituted for theta (θ) and the equation would change to:

$$v = ASin\left(\tfrac{t}{P} \times 360\right)$$

Given $\frac{1}{P} = f$, the statement $\left(\frac{t}{P} \times 360\right)$ in the expression above can be rewritten as $(t \times f \times 360)$. Substituting this changes the expression above to $v = ASin(tf360)$ which makes it a little easier to use. Using *angular frequency* (ω - omega - which is measured in radians) instead of frequency in Hertz can make it easier still. Angular frequency is measured in radians where one radian is equal to 2π so $\omega = 2\pi f$. Substituting omega into the expression above changes it to:

$$v = ASin\omega t$$

Note: To solve for v in this expression, your calculator must be set to *radians* mode instead of degrees.

This expression is the standard form used for mathematical representations of complex waveforms where each sinewave in the waveform is represented by one of these statements. For an example, consider the complex waveform in Figure 1 below. It is formed by adding the two sinewaves shown where the second sinewave is three times the frequency of the first but a third of its voltage.

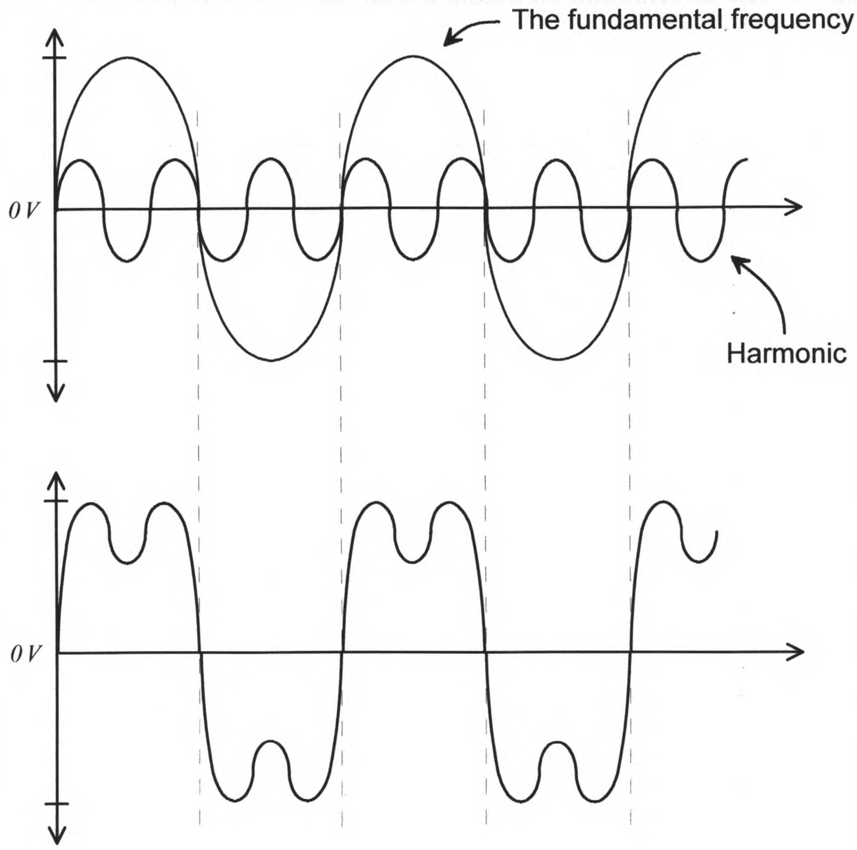
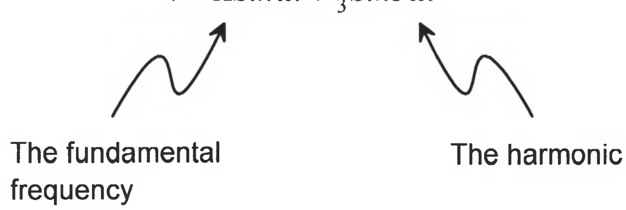


Figure 1

The expression that describes the result is:

$$v = ASin\omega t + \tfrac{A}{3}Sin3\omega t$$



Notice that the expression for the harmonic ($v = \frac{4}{3}Sin3\omega t$) reflects the fact that its peak voltage is a third of the fundamental voltage and its frequency is three times the frequency of the fundamental.

Expressing complex waveforms using a mathematical expression in this way is useful because, if we know the frequency of the fundamental waveform and it's peak voltage we can calculate the waveform's instantaneous voltage at any point in the cycle. That said, you'll not be asked to do so for this module. The information on mathematical models is included here for your interest only.

Appendix 2

The harmonic content of a sawtooth wave

One of the sawtooth waves is made up of the fundamental frequency and all (whole number) harmonics each having a peak voltage that is proportionally smaller than the fundamental. Again, the harmonics must be in phase with the fundamental. Consider a 1kHz 1Vp-p sawtooth wave for example. It consists of the following sinewaves:

Frequency components	Electrical properties	
Fundamental	1kHz	1Vp-p
Second harmonic	2kHz	0.5Vp-p
Third harmonic	3kHz	0.333Vp-p
Fourth harmonic	4kHz	0.25Vp-p
Fifth harmonic	5kHz	0.2Vp-p
Sixth harmonic	6kHz	0.167Vp-p
Seventh harmonic	7kHz	0.143Vp-p
Eighth harmonic	8kHz	0.125Vp-p

. . . and so on to infinity

The mathematical representation of the triangular wave is:

$$v = ASin\omega t + \tfrac{A}{2}Sin2\omega t + \tfrac{A}{3}Sin3\omega t + \tfrac{A}{4}Sin4\omega t + \tfrac{A}{5}Sin5\omega t + \ldots$$

. . . for all odd and even numbers to infinity.

Appendix 3

Section summaries

The theory notes in each Section of this workbook are designed to help you learn about the fundamentals of AC theory and filters. But when you think about it, there's a difference between learning and remembering. These section summaries are designed to help you remember the theory.

Section 1 (9136C - Part 2)

Objectives and Summary

1. Define the terms *complex waveform, fundamental frequency, harmonics* and *harmonic distortion*

   A complex waveform is any signal that is not a pure sinewave. This usually means that the signal consists of many sinewaves.

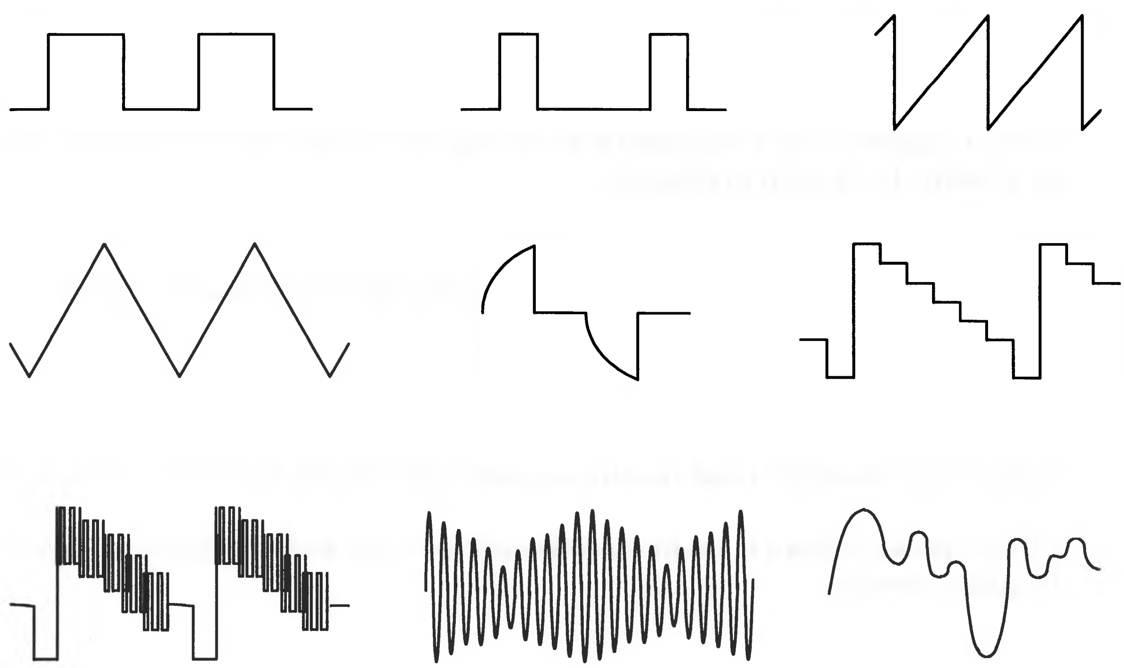
   The fundamental frequency is the sinewave in a complex waveform that has the same frequency as the signal itself.

   The harmonics are all the sinewaves in a complex waveform other than the fundamental.

   Distortion changes the shape of a waveform. In the process, the distortion introduces sinewaves that were not originally present in the original signal. Hence distortion is more accurately called harmonic distortion.

2. Recognise a complex waveform

   Examples include:



3. Describe the difference between repetitive waveforms represented in the time and frequency domains

   The difference is the X-axis. The X-axis of signals drawn in the time domain is time and so they look like what you would see on a CRO. The X-axis of signals drawn in the frequency domain is frequency. So, instead of seeing what the signal looks like, you see what its made-up of.

4. List the frequency components present in a squarewave given its frequency

   Where the squarewave's frequency is *fsq* and its amplitude is *Vsq*

Frequency components	**Electrical properties**
Fundamental	$1 \times fsq$ & Vsq
Third harmonic	$3 \times fsq$ & $Vsq \div 3$
Fifth harmonic	$5 \times fsq$ & $Vsq \div 5$
Seventh harmonic	$7 \times fsq$ & $Vsq \div 7$
Ninth harmonic	$9 \times fsq$ & $Vsq \div 9$
Eleventh harmonic	$11 \times fsq$ & $Vsq \div 11$
Thirteenth harmonic	$13 \times fsq$ & $Vsq \div 13$

. . . and so on to infinity.

5. State the usefulness of the square wave as a test signal for audio systems

   Using a squarewave as a test signal lets you qualitatively compare the frequency response of an amplifier to a known working unit.

6. Explain the difference between signals shown in the time and frequency domains

   See "3" above.

7. Name the test equipment that visually represent signals in the time and frequency domains

   CROs represent signals in the time domain and spectrum analysers represent signals in the frequency domain.

Section 2 (9136C - Part 2)

Objectives and Summary

1. Explain the purpose of filters in electronic systems

   To direct sinewaves at certain frequencies to the correct part of the circuit while blocking the other sinewaves.

2. List practical applications for low-pass, high-pass, band-pass and band-stop filters

   Low-pass:

   - passive crossover for woofer output
   - bass control of a hi-fi system
   - ripple filter in a DC power supply

   High-pass:

   - passive crossover for tweeter output
   - treble control of a hi-fi system
   - rumble filter on a microphone input

   Band-pass:

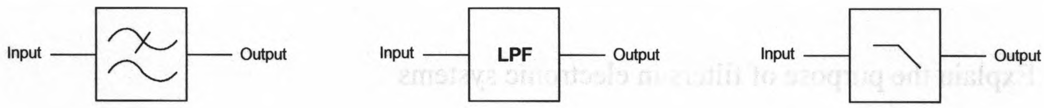
   - passive crossover for mid-range output
   - in a *graphic equaliser* to "boost" or "cut" selected bands of frequencies
   - tuning circuits in communications equipment like radio & tv

   Band-stop

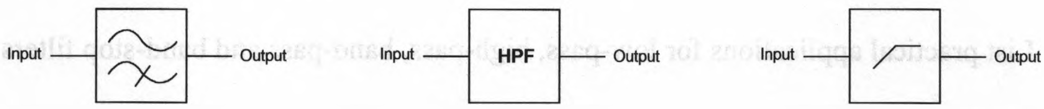
   - a scratch filter on record players
   - to remove noise at certain frequencies from signals that would interfere with normal operation of communications systems

3. Draw schematic symbols for a low-pass, high-pass, band-pass and band-stop filter

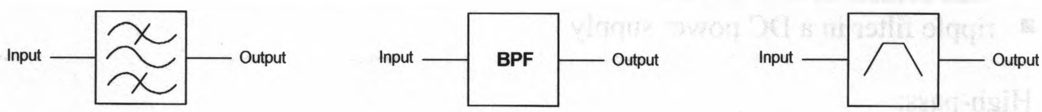
Low-pass:



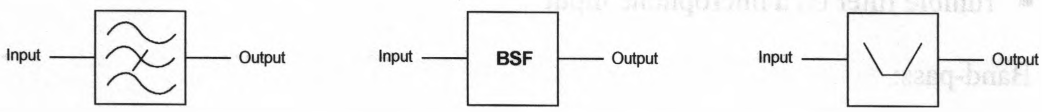
High-pass:



Band-pass:



Band-stop:



4. Explain the operation of a low-pass, high-pass, band-pass and band-stop filter module with respect to their frequency response

Low-pass filters allow sinewaves below a set frequency to pass from input to output relatively unaffected but attenuate sinewaves above the set frequency.

High-pass filters allow sinewaves above a set frequency to pass from input to output relatively unaffected but attenuate sinewaves at frequencies below the set frequency.

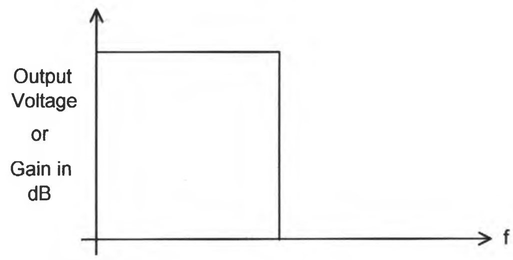
Band-pass filters allow sinewaves between two set frequencies to pass through relatively unaffected but attenuate sinewaves at frequencies above or below this.

Band-stop filters attenuate sinewaves between two set frequencies but allow sinewaves at frequencies above and below these to pass from input to output relatively unaffected.

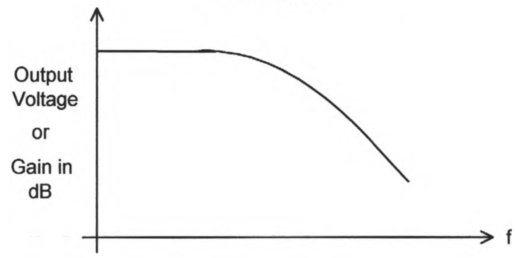
5. Draw the ideal and practical frequency response of a low-pass, high-pass, band-pass and band-stop filter

Low-pass:

Ideal

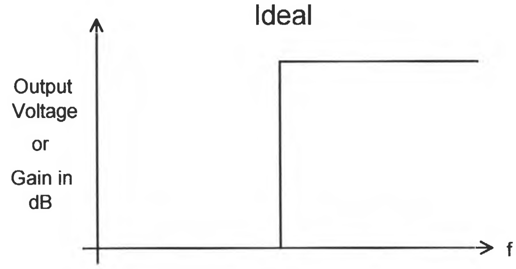


Practical

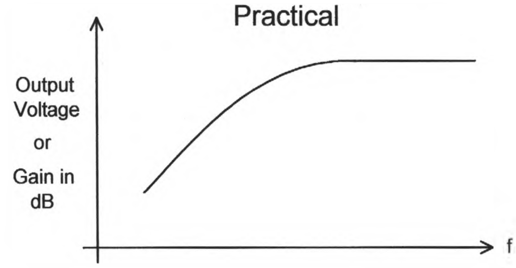


High-pass:

Ideal

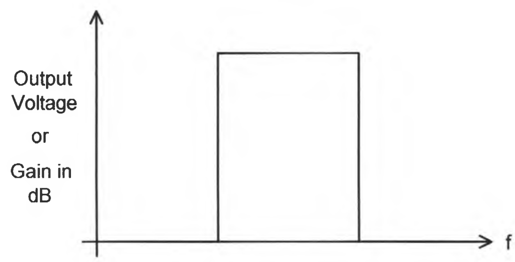


Practical

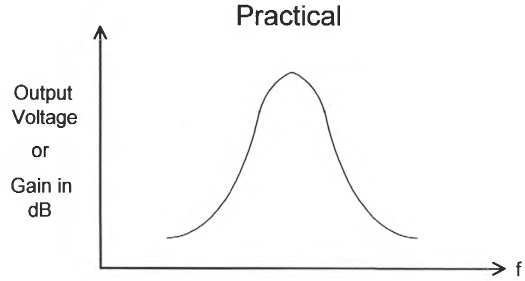


Band-pass:

Ideal

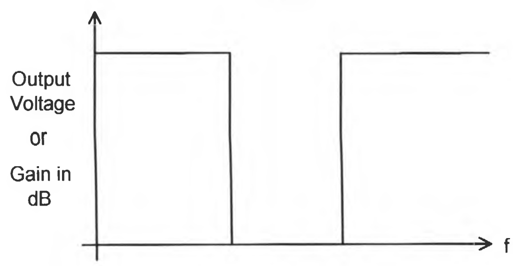


Practical

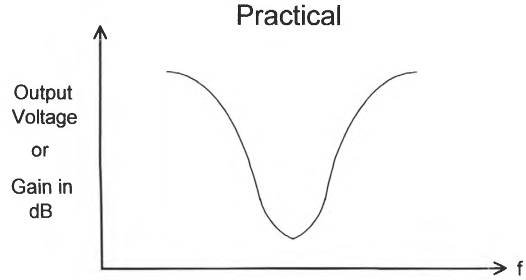


Band-stop:

Ideal



Practical



Section 3 (9136C - Part 2)

Objectives and Summary

1. Define terms commonly used to describe the attributes and performance characteristics of filters

   Gain: The numerical comparison between a filter's input and output voltages. Usually, the output is smaller than the input so the gain is actually a loss but we can still call it "gain".

   Roll-off: The region of a low-pass & high-pass filter's frequency response where its gain (and output voltage) drops as the frequency changes.

   Skirt: Same as above for band-pass and band-stop filter's frequency response but there's two of them instead of one.

   Slope: The "steepness" of a filter's roll-off or skirts. The figures you must remember are 6dB/octave or 20dB/decade for 1st order filters.

   Cut-off frequency: The frequency where the filter's gain has dropped to 0.707 of the pass-band's maximum gain (as a ratio). Or to where the filter's gain has dropped by 3dB from the pass-band's maximum gain (in decibels). BPFs and BSFs have two cut-off frequencies and so they're called *f1* and *f2* instead.

   Pass-band: The region on a filter's frequency response where its gain (and therefore the output voltage) is maximum and is relatively flat.

   Stop-band: The region on a band-stop filter's frequency response where its gain (and therefore the output voltage) is attenuated by 3dB or more (relative to the gain in the pass-band).

   Bandwidth: The frequency width of its pass-band. Bandwidth can be calculated from the *f1* and *f2* frequencies using the equation: $BW = f2 - f1$.

   Centre frequency: The frequency of band-pass filters with the highest gain. The centre frequency can be calculated from the *f1* and *f2* frequencies using the equation: $fc = \sqrt{f1 \times f2}$.

   Filter order: This refers to the number of capacitors and inductors (known in filter terminology as *reactive elements*) that the filter is made from. A filter with only one reactive element is a 1st order filter. A filter with two reactive elements is a 2nd order filter and so on.

   Insertion loss: This is the name that is often used for a filter's pass-band gain when it's output is smaller than its input (so attenuated). Put another way, its when the pass-band gain (in decibels) is a negative number.

Pass-band amplification: This is the name that is often used for a filter's pass-band gain when the output is bigger than the input. Put another way, its when the pass-band gain (in decibels) is greater than 0dB.

2. **Calculate the output voltage of a filter given the amplitude and frequency of the input voltage, the type of filter, the filter's slope and its insertion loss or amplification**

Use the review questions to practise this. The equation is:

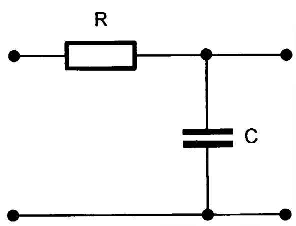
$Av_{(dB)} = 20Log\frac{Vout}{Vin}$

Section 4 (9136C - Part 2)

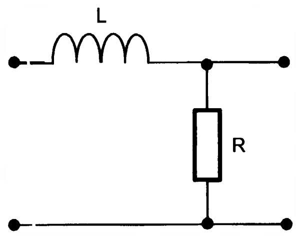
Objectives and Summary

1. Identify passive RC and RL low-pass and high-pass filter circuits

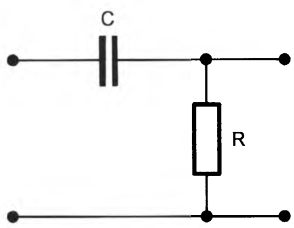
Passive RC LPF



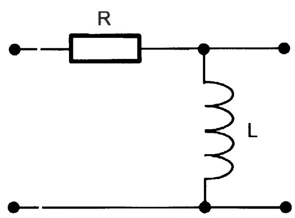
Passive RL LPF



Passive RC HPF



Passive RL HPF



2. Specify the slope of the roll-off of passive first order RC and RL low-pass and high-pass filter circuits

   6dB/octave or 20dB/decade (It's the same thing only the scale is different).

3. State the relative phase relationship between input and output voltages at a given frequency for passive RC and RL low-pass and high-pass filter circuits

   For frequencies in the pass-band: The input and output voltages are in-phase for both types of filter.

   For frequencies in the roll-off region: The output lags the input for LPFs. The output leads the input for HPFs.

4. Specify the phase relationship between input and output voltages at the cut-off frequency for passive RC and RL low-pass and high-pass filter circuits

   LPFs: -45°

   HPFs: +45°

5. Calculate the cut-off frequency of passive first order RC and RL low-pass and high-pass filter circuits

   Use the review questions to practise this. The equations are:

   For RC LPF & HPF: $fc = \frac{1}{2\pi RC}$

   For RL LPF & HPF: $f_c = \frac{R}{2\pi L}$

Section 5 (9136C - Part 2)

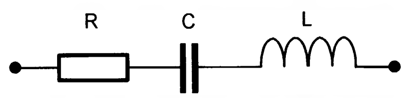
Objectives and Summary

1. Define resonance as it relates to electrical circuits

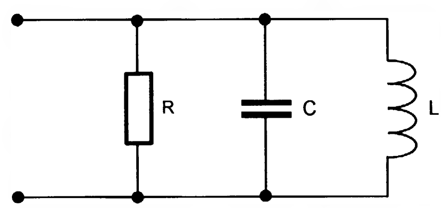
   Resonance is the effect whereby a stimulus of a given amount causes a much bigger reaction at one frequency than any other frequency.

2. Draw a series and parallel RLC circuit

Series



Parallel



3. Describe the frequency characteristics of series and parallel RLC circuits

   Series:

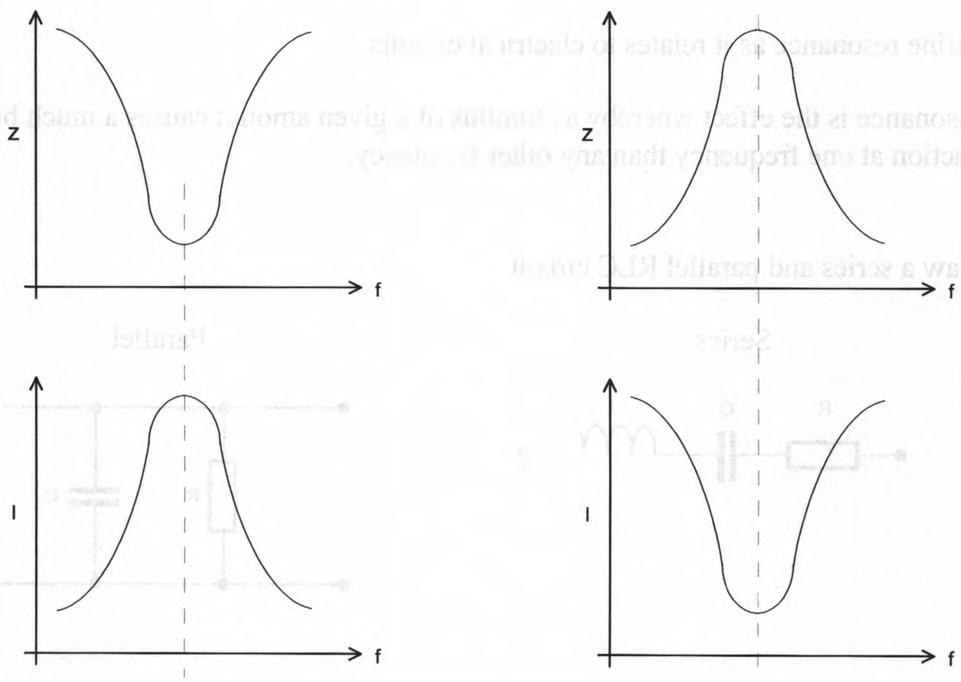
   If the frequency of the input signal sweeps from very low to very high, X_L increases and X_C decreases. The net effect of this is, Z decreases reaching a minimum at resonance then increases again.

   Parallel:

   If the frequency of the input signal sweeps from very low to very high, X_L increases and X_C decreases. The net effect of this is, Z increases reaching a maximum at resonance then decreases again.

4. Draw the graphs of current and total circuit impedance versus frequency for series and parallel RLC circuits

Series | Parallel



5. Describe the conditions of series and parallel RLC circuits at the resonant frequency

Series:

- X_C and X_L are equal and opposite values cancelling each other out
- The total circuit impedance is purely resistive
- The total circuit impedance is minimum
- The circuit current is maximum

Parallel

- X_C and X_L are equal and opposite values cancelling each other out
- The total circuit impedance is purely resistive
- The total circuit impedance is maximum
- The circuit current is minimum

6. Calculate the resonant frequency of series and parallel RLC circuits

Use the review questions to practise this. The equation is:

$$fr = \frac{1}{2\pi\sqrt{LC}}$$

7. Calculate the voltage across the components in an RLC circuit for a given frequency

Use the review questions to practise this. The procedure and equations are:

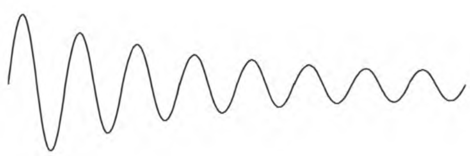
- Find X_C and X_L at the nominated frequency using $X_C = \frac{1}{2\pi f C}$ and $X_L = 2\pi f L$
- Find the total circuit impedance using $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- Find the circuit current using $I = \frac{E}{Z}$
- Find the voltage across the individual components using $V_C = I \times X_C$, $V_L = I \times X_L$ and $V_R = I \times R$

8. Explain the terms *voltage magnification effect*, *Q-factor* and *damped oscillation*

Voltage magnification: This applies only to RLC circuits. It occurs when the potential difference across the capacitor and/or inductor at certain frequencies is bigger than the applied voltage.

Q-factor: A numerical indication of an RLC (or tuned) circuit's usefulness for a particular application.

Damped oscillation: As a picture is worth a thousand words...



9. Calculate the *Q-factor* of an RLC circuit given all component values

Use the review questions to practise this. The equation is:

Series RLC circuits: $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

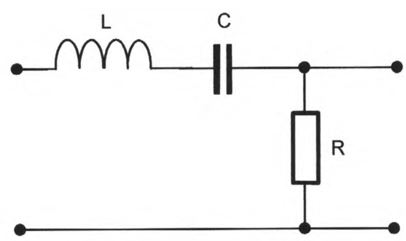
Parallel RLC circuits: $Q = R\sqrt{\frac{C}{L}}$

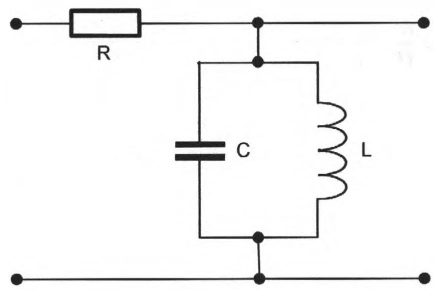
Section 6 (9136C - Part 2)

Objectives and Summary

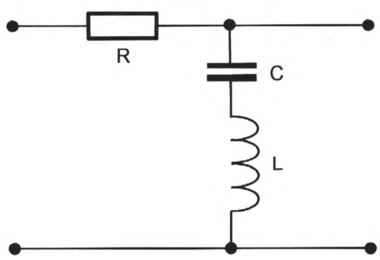
1. Identify passive RCL band-pass and band-stop filter circuits

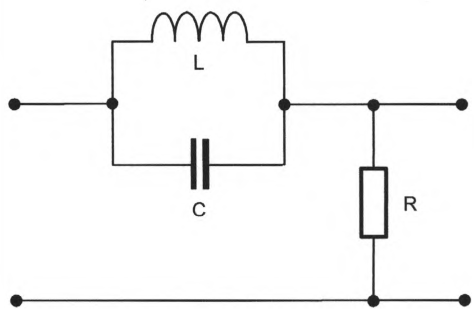
Passive band-pass





Passive band-stop





2. Calculate the centre frequency of passive RCL band-pass and band-stop filter circuits

Use the review questions to practise this. The equation is:

$$fc = \frac{1}{2\pi\sqrt{LC}}$$

3. State the phase relationship in degrees between input and output voltages at the centre frequency for passive RCL band-pass and band-stop filter circuits

In the pass-band the input and output voltages are in-phase.

Outside the pass-band there is a phase shift. You'll not be asked to calculate the amount and polarity of this phase shift.

4. Define the term *selectivity*

Selectivity refers to how the bandwidth and steepness of the skirts of band-pass filters (and band-stop) facilitate rejection adjacent nearby signals.

5. Calculate the centre frequency, bandwidth or Q-factor of a band-pass and band-stop filter given any two of these parameters

Use the review questions to practise this. The equation is:

$$BW = \frac{fc}{Q}$$

Answers to Review Questions

Section 1

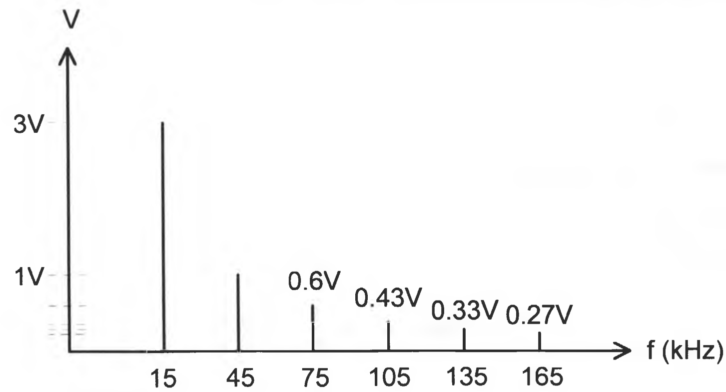
1. Spectrum analyser

2. 12kHz, 850mVp-p sinewave

3. 36kHz, 283.33mVp-p sinewave

4. 60kHz, 170mVp-p sinewave

5.



Section 2

1. High-pass filter

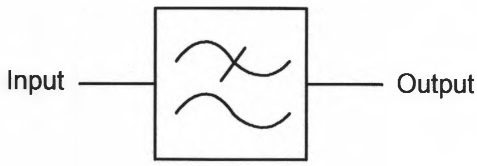
2. Band-stop filter

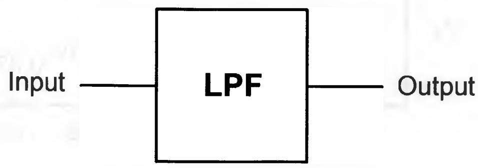
3. Band-pass filter

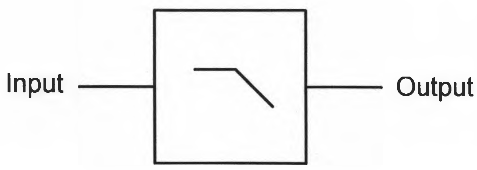
4. Notch filter

5. The frequency response of an ideal filter allows all wanted signals to pass through unaffected and rejects all unwanted signals totally. Practical filters let some of the unwanted signals pass through to the output (though they'll be attenuated).

6. For the low-pass filter you should have drawn one of the following:

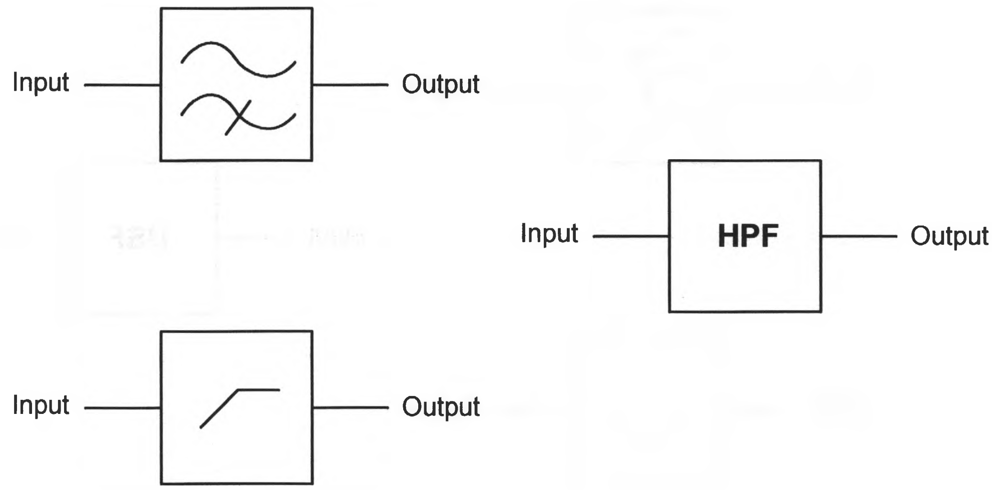




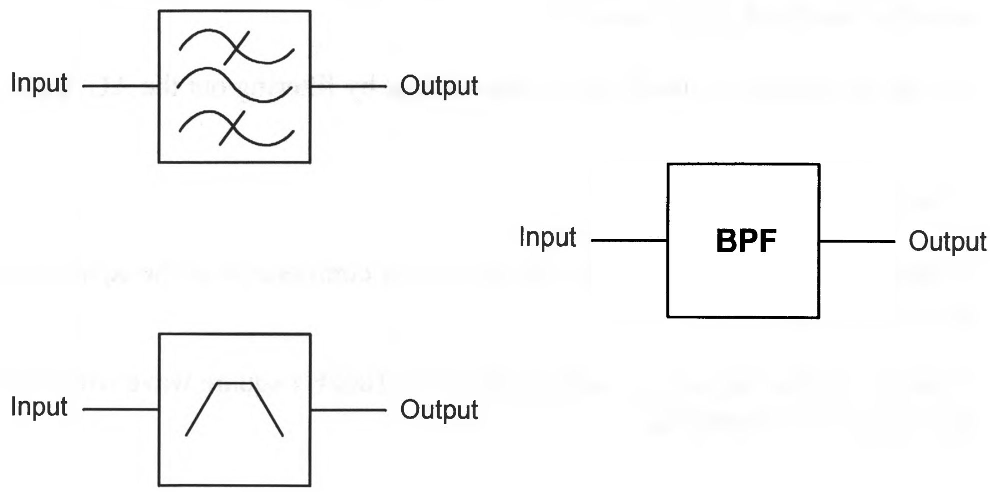


Section 2 (continued)

For the high-pass filter you should have drawn one of the following:

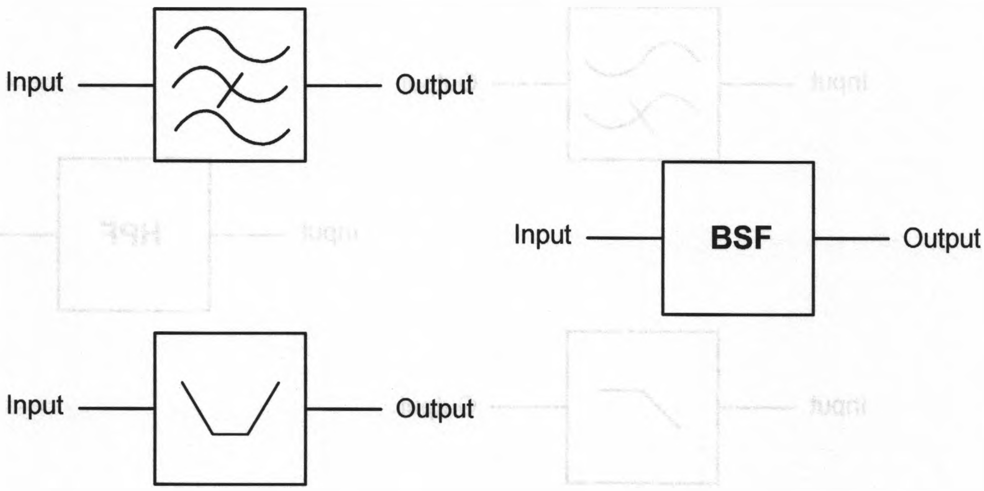


For the band-pass filter you should have drawn one of the following:



Section 2 (continued)

For the band-stop filter you should have drawn one of the following:



7. The *crossover* in a speaker box to direct lower frequency signals to the woofer and sub-woofer.

   The bass control circuit in a hi-fi system used to vary the amount of lower frequencies amplified and sent to the speakers.

   In a power supply to smooth the output voltage by filtering out the AC ripple voltage.

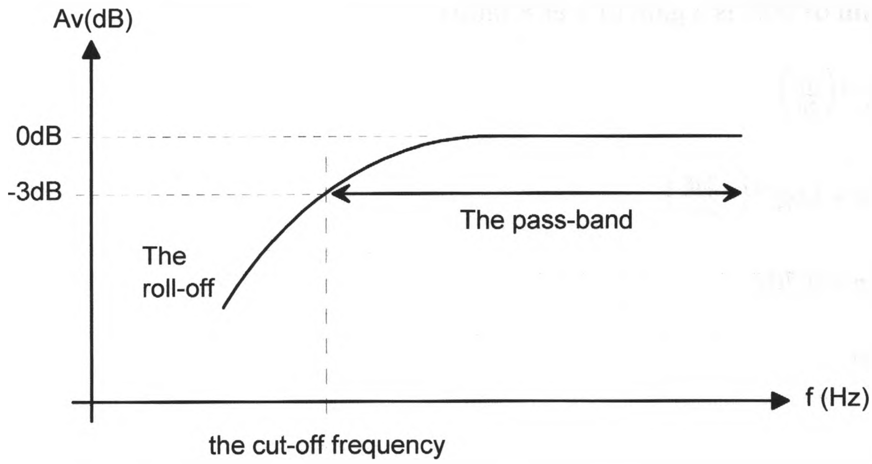
8. A band-pass filter

9. A 50kHz, 5Vp-p sinewave. All other frequency components of the square wave would be rejected by the filter.

10. Nothing. All the frequency components of the 100kHz square wave would be rejected including the fundamental.

Section 3

1.



2. frequencies in the pass-band.
3. filter's gain goes down.
4. Active high-pass filter
5. 20dB/decade
6. 20dB
7. Approximately 3.5kHz
8. Approximately 450Hz
9. First order.
10. Low-pass filter.
11. -8.2dB
12. $Vout = Vin \times Log^{-1}\left(\frac{-AvdB}{20}\right)$ Or $Vout = 3.5V \times 0.707$

$Vout = 9V \times Log^{-1}\left(\frac{-11.2dB}{20}\right)$ $Vout = 2.48V$

$Vout = 9V \times 0.2754$

$Vout = 2.48V$

Section 3 (continued)

13. 0dB (250kHz is a frequency in this filter's passband and the passband gain is given as 0dB)

14. 850mVp-p (A gain of 0dB is a gain of 1 as a ratio)

15. $Vout = Vin \times Log^{-1}\left(\frac{Av}{20}\right)$

$Vout = 850mVp\text{-}p \times Log^{-1}\left(\frac{-3dB}{20}\right)$

$Vout = 850mVp\text{-}p \times 0.707$

$Vout = 601mVp\text{-}p$

16. -32dB

Note: It is stated that this is a first order filter and so it has a slope of 20dB per decade. It is also stated that the gain at 50kHz is -12dB. 5kHz is one decade further down the slope from 50kHz so the gain at 5kHz is 20dB down from -12dB.

17. $Vout = Vin \times Log^{-1}\left(\frac{Av}{20}\right)$

$Vout = 850mVp\text{-}p \times Log^{-1}\left(\frac{-32dB}{20}\right)$

$Vout = 21.31mVp\text{-}p$

18. Band-pass filter

19. 3500Hz

20. $fc = \sqrt{f1 \times f2}$

$fc = \sqrt{300 \times 3800}$

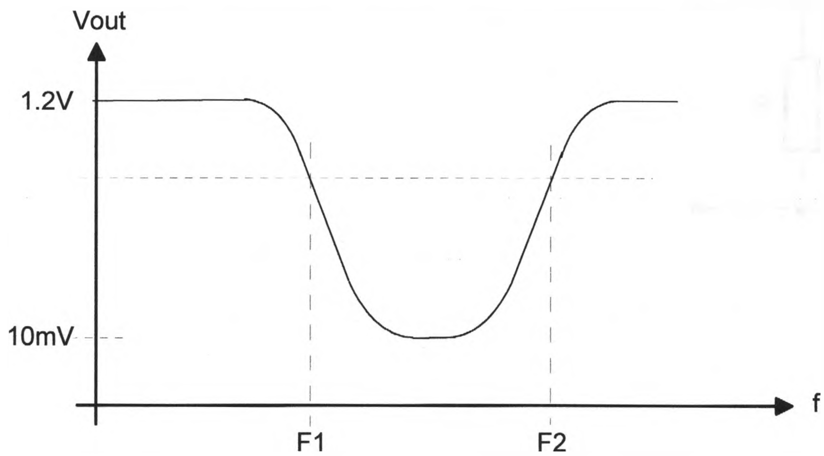
$fc = 1067.7Hz$

Note: You can't use the arithmetic mean to answer this question because the filter's Q-factor is less than 10. Q-factor is explained in Section 5.

Section 3 (continued)

21. Band-stop filter

22.



23. frequencies outside the stop-band.

24. 0.85V

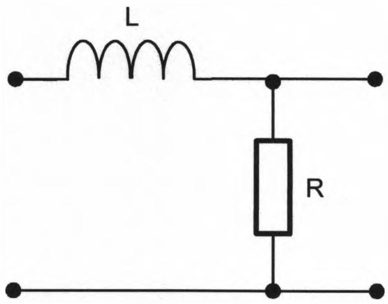
25. $Av = 20Log\left(\frac{Vout(\text{min})}{Vout(\text{max})}\right)$

$Av = 20Log\left(\frac{10mV}{1.2V}\right)$

$Av = -41.6dB$

Section 4

1.



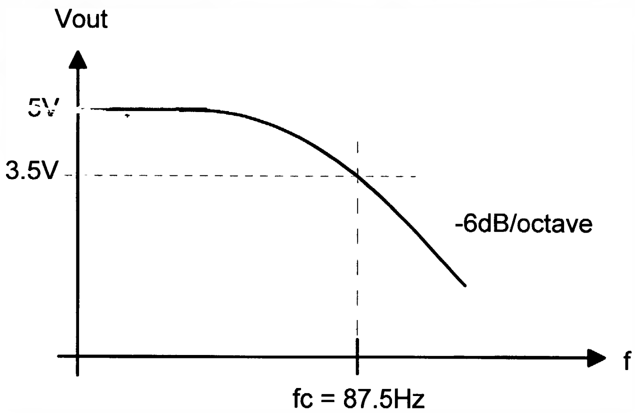
2. $f_c = \frac{R}{2\pi L}$

$f_c = \frac{2.2k\Omega}{2\pi \times 4}$

$f_c = 87.5Hz$

3. -45°

4.



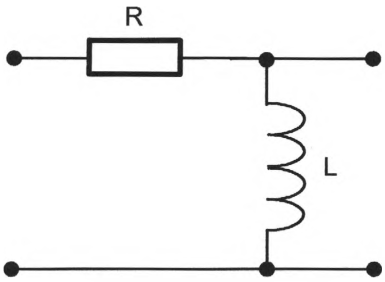
5. $L = \frac{R}{2\pi f_c}$

$L = \frac{2.2k\Omega}{2\pi \times 12kHz}$

$L = 29.2mH$

Section 4 (continued)

6.



7. 5.22kHz

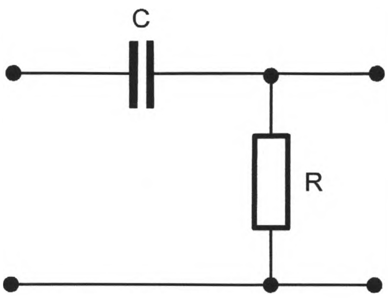
8. +45°

9. $R = 2\pi L f_c$

$R = 2\pi \times 250mH \times 40kHz$

$R = 62.8k\Omega$

10.



11. $fc = \frac{1}{2\pi RC}$

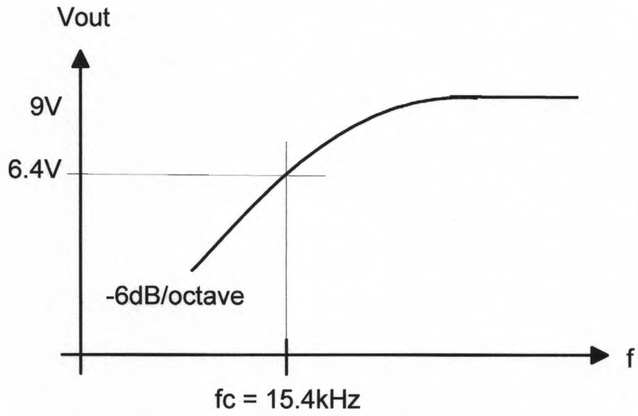
$fc = \frac{1}{2\pi \times 4.7k\Omega \times 2.2nF}$

$fc = 15.4kHz$

12. +45°

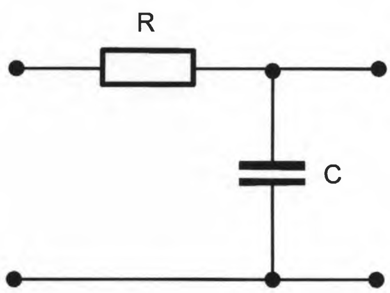
Section 4 (continued)

13.



14. The cut-off frequency would decrease.

15.



16. 48.23kHz

17. -45°

18. $C = \frac{1}{2\pi Rf}$

$C = \frac{1}{2\pi \times 10k\Omega \times 100kHz}$

$C = 159pF$

19. The cut-off frequency would decrease.

Section 5

1. purely resistive and minimum.

2. capacitive.

3. inductive.

4. maximum.

5. $fr = \frac{1}{2\pi\sqrt{LC}}$

$fr = \frac{1}{2\pi\sqrt{330mH \times 680pF}}$

$fr = 10.62kHz$

6. 115Ω

7. 26.09mAp-p

8. $X_L = 2\pi f L$

$X_L = 2\pi \times 10.62kHz \times 330mH$

$X_L = 22.02k\Omega$

$V_L = I_L \times X_L$

$V_L = 26.09mAp\text{-}p \times 22.02k\Omega$

$V_L = 574.5Vp\text{-}p$

9. 3Vp-p

10. $Q = \frac{|V_L|}{|V_R|}$

$Q = \frac{574.5Vp\text{-}p}{3Vp\text{-}p}$

$Q = 191.5$

Section 5 (continued)

11. $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$Q = \frac{1}{115\Omega}\sqrt{\frac{330mH}{680pF}}$

$Q = 191.6$

12. Transposing $fr = \frac{1}{2\pi\sqrt{LC}}$ to make C the subject give:

$C = \frac{\left(\frac{1}{2\pi fr}\right)^2}{L}$ Note: if you can't transpose this equation, you should see the teacher.

$C = \frac{\left(\frac{1}{2\pi \times 75kHz}\right)^2}{4.2mH}$

$C = 1.07nF$

13. purely resistive and maximum.

14. inductive.

15. capacitive.

16. minimum.

17. $fr = \frac{1}{2\pi\sqrt{LC}}$

$fr = \frac{1}{2\pi\sqrt{70\mu H \times 22pF}}$

$fr = 4.056MHz$

18. 4k7Ω

19. 2.63 Note: For parallel circuits use: $Q = R\sqrt{\frac{C}{L}}$

Section 5 (continued)

20. Transposing $fr = \frac{1}{2\pi\sqrt{LC}}$ to make C the subject give:

$$L = \frac{\left(\frac{1}{2\pi fr}\right)^2}{C}$$

Note: if you can't transpose this equation, you should see the teacher.

$$L = \frac{\left(\frac{1}{2\pi\times 630kHz}\right)^2}{390pF}$$

$$L = 163.6\mu H$$

21. Tank circuit

Section 6

1. The higher the Q-factor, the more selective the filter.

2. The higher the Q-factor the smaller the bandwidth.

3. The higher the Q-factor, the steeper the skirts.

4. Increase the Q (by reducing the DC resistance in the circuit)
   Use filter designs with more inductors and capacitors.

5. Band-pass filter

6. Because at relatively low and high frequencies, the total reactance of the series LC combination is very high and has almost all of the input voltage dropped across it. This means that there is almost no voltage dropped across the resistor.

7. Because at the centre frequency, the reactances of the inductor and capacitor are equal and opposite cancelling each other out. This means that all of the input voltage is dropped across the resistor.

8. 1.54kHz

9. 63

10. $BW = \frac{fr}{Q}$

    $BW = \frac{1.54kHz}{63}$

    $BW = 24.4Hz$

11. 0°

12. 500mAp-p

13. 3V

14. Band-stop filter

15. Because at relatively low and high frequencies, the total reactance of the parallel LC combination is very low and has almost none of the input voltage dropped across it. This means that almost all of the input voltage is developed across the resistor.

Section 6 (continued)

16. Because at the centre frequency, the total reactance of the parallel LC combination is maximum. This means that the LC combination has most of the input voltage developed across it leaving almost none for the resistor.

17. 987kHz

18. 58.3 (Be careful to use the correct equation here - It's a parallel circuit.)

19. 16.93kHz

20. 0°

21. $Q = \frac{fr}{BW}$

$Q = \frac{987kHz}{5kHz}$

$Q = 197.4$